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Finding Common Weights in Two-Stage Network DEA

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Abstract

In data envelopment analysis (DEA), multiplier and envelopment CCR models evaluate the decision-making units (DMUs) under optimal conditions. Therefore, the best prices are allocated to the inputs and outputs. Thus, if a given DMU was not efficient under optimal conditions, it would not be considered efficient by any other models. In the current study, using common weights in DEA, a number of decision-making units are evaluated under the same conditions, and a number of two-stage network DEA models are proposed within the framework of multi-objective linear programming (MOLP) for finding common weights. Furthermore, using the infinity norm, common weight sets are determined in two-stage network models with MOLP structures.

Keywords: data envelopment analysis, common weights, ranking, two-stage network, decision-making unit.

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1. Introduction

Data envelopment analysis (DEA) is a method for measuring the performance of decision-making units (DMUs) that consume multiple inputs to produce multiple outputs. Two basic DEA models, namely CCR and BCC, have become the standard for performance evaluation under assumptions of constant (CRS) and variable returns to scale (VRS). DEA often deals with single-stage production processes in which the internal structure of DMUs is not taken into account. On the other hand, network DEA involves multi-stage processes, where the basic structure, which shows the trend of intermediate measures in-between stages, plays a critical role. Fare and Grosskopf (1996) were among the first to study efficiency in such processes within the framework of a model for analyzing network activity. Castelli et al. (2010) provided a comprehensive and categorized overview of the models and methods developed for various configurations of multi-stage production. Kao (2014), presented a full categorization of the literature on data envelopment analysis based on network structure type and applied model, where the same weights were allocated to intermediate measures, regardless of whether the measures were considered the outputs of the first stage or the inputs of the second stage. Liang et al. (2008) and Cook et al. (2010) studied efficiency evaluation in two-stage processes using theoretical concepts. Kao et al. (2014) used a multi-objective programming method for efficiency evaluation in network structures. Recently, Despotis et al. (2014) introduced a combined method of efficiency measurement in two-stage networks, in which the efficiency of each stage was estimated first, and then the overall efficiency was obtained on that basis. However, the weakness of this method is that it cannot be easily extended to multi-stage network processes. The current research focuses on the different orientations selected for stages one and two of a two-stage network, which are technically created in order to simplify the models and keep them within the field of linear programming. In this study, two-stage network processes are discussed with focus on a variety of distinct processes covering all possible configurations.

2. Preliminaries

This section provides the basic concepts of the two-stage network in data envelopment analysis.

Consider n decision-making units, denoted by the subscript j (DMU_j), that use m input vectors X_j for consumption in the first stage. The vector Z_j with q elements is used as the output of stage one and the input of stage two in the DEA network.

$Y_j=(y_{rj}\ , r=1,\ldots,s)$: Vector of final outputs in DMU_j with the weight vector $U=(u_1,\ldots,u_s)$.

$L_j=(l_{dj}\ , d=1,\ldots,a)$: Vector of external inputs in DMU_j with the weight vector $H=(h_1,\ldots,h_a)$.

$K_j=(k_{cj}\ , c=1,\ldots,b)$: Vector of external outputs in DMU_j with the weight vector $T=(t_1,\ldots,t_b)$.

e_j^o: Overall efficiency of DMU_j

e_j^1: Efficiency of stage one in DMU_j

e_j^o: Efficiency of stage two in DMU_j

Consider the case where we have X inputs, Y outputs, and Z intermediate measures, with no external inputs or outputs. In this case, the efficiency of stages one and two are defined as follows:

$$e_j^1=\frac{\sum_{p=1}^{q} z_{pj}w_p}{\sum_{i=1}^{m} x_{ij}v_i}\quad ,\quad e_j^2=\frac{\sum_{r=1}^{s} y_{rj}u_r}{\sum_{p=1}^{q} z_{pj}w_p}$$

and the overall efficiency of DMU_j is calculated as:

$$e_j^2=\frac{\sum_{r=1}^{s} y_{rj}u_r}{\sum_{i=1}^{m} x_{ij}v_i}.$$

The following shows the DEA model for efficiency measurement in the first network stage:

$$\begin{aligned}
&\text{Max}\ \frac{\sum_{p=1}^{q} z_{po} w_p}{\sum_{i=1}^{m} x_{io} v_i}\\
&\text{s.t:}\quad \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0\ ,\\
&j = 1, \dots, n\\
&\quad v_i \ge \varepsilon\ ,\ \text{i}=1,\dots,\text{m}\\
&\quad w_p \ge \varepsilon\ ,\ \text{p}=1,\dots,\text{q}
\end{aligned} \tag{1}$$

The DEA model for efficiency evaluation in the second network stage is formulated as follows:

$$\begin{aligned}
&\text{Max}\ \frac{\sum_{r=1}^{s} y_{ro} u_r}{\sum_{p=1}^{q} z_{po} w_p}\\
&\text{s.t:}\quad \sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0\ ,\\
&j = 1, \dots, n\\
&\quad u_r \ge \varepsilon\ ,\ \text{r}=1,\dots,\text{s}\\
&\quad w_p \ge \varepsilon\ ,\ \text{p}=1,\dots,\text{q}
\end{aligned} \tag{2}$$

By adding the restriction of Model (1) to Model (2) and vice versa, we will arrive at Models (3) and (4), respectively (Despotis et al., 2014).

$$\begin{aligned}
&\text{Max}\ \frac{\sum_{p=1}^{q} z_{po} w_p}{\sum_{i=1}^{m} x_{io} v_i}\\
&\text{s.t:}\quad \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0\ ,\\
&j = 1, \dots, n\\
&\sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0\ ,\\
&j = 1, \dots, n\\
&\quad v_i \ge \varepsilon\ ,\ \text{i}=1,\dots,\text{m}\\
&\quad u_r \ge \varepsilon\ ,\ \text{r}=1,\dots,\text{s}\\
&\quad w_p \ge \varepsilon\ ,\ \text{p}=1,\dots,\text{q}
\end{aligned} \tag{3}$$

The linear form of Model (3) is as follows:

$$\begin{aligned}
&\text{Max}\ \sum_{p=1}^{q} z_{po}\, w_p\\
&\text{s.t:}\quad \sum_{i=1}^{m} x_{io}\, v_i = 1\\
&\qquad \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0\ ,\\
&\text{j}=1,\dots,\text{n}\\
&\qquad \sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0\ ,\\
&\text{j}=1,\dots,\text{n}\\
&\quad v_i \ge \varepsilon\ ,\ \text{i}=1,\dots,\text{m}\\
&\quad u_r \ge \varepsilon\ ,\ \text{r}=1,\dots,\text{s}\\
&\quad w_p \ge \varepsilon\ ,\ \text{p}=1,\dots,\text{q}
\end{aligned} \tag{4}$$

Model (5) is used to calculate efficiency in the second stage (Despotis et al., 2014).

$$\begin{aligned}
&\text{Max}\ \frac{\sum_{r=1}^{s} y_{ro} u_r}{\sum_{p=1}^{q} z_{po} w_p}\\
&\text{s.t:}\quad \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0\ ,\\
&j = 1, \dots, n\\
&\qquad \sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0\ ,\\
&j = 1, \dots, n\\
&\quad v_i \ge \varepsilon\ ,\ \text{i}=1,\dots,\text{m}\\
&\quad u_r \ge \varepsilon\ ,\ \text{r}=1,\dots,\text{s}\\
&\quad w_p \ge \varepsilon\ ,\ \text{p}=1,\dots,\text{q}
\end{aligned} \tag{5}$$

Model (6) is the linear form of Model (5).

$$\begin{aligned}
&\text{Max}\ \sum_{r=1}^{s} y_{ro}\, u_r\\
&\text{s.t:}\quad \sum_{p=1}^{q} z_{po}\, w_p = 1\\
&\qquad \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0\\
&\text{j}=1,\dots,\text{n}\\
&\qquad \sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0\ ,\\
&\text{j}=1,\dots,\text{n}\\
&\quad v_i \ge \varepsilon\ ,\ \text{i}=1,\dots,\text{m}\\
&\quad u_r \ge \varepsilon\ ,\ \text{r}=1,\dots,\text{s}\\
&\quad w_p \ge \varepsilon\ ,\ \text{p}=1,\dots,\text{q}
\end{aligned} \tag{6}$$

Models (3) and (5) have common restrictions; thereby, the following can be formulated: (see Despotis et al. (2016))

$$\begin{aligned}
&\text{Max}\ \frac{\sum_{p=1}^{q} z_{po} w_p}{\sum_{i=1}^{m} x_{io} v_i}\\
&\text{Max}\ \frac{\sum_{r=1}^{s} y_{ro} u_r}{\sum_{p=1}^{q} z_{po} w_p}\\
&\text{s.t:}\quad \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0\ ,\\
&j = 1, \dots, n\\
&\qquad \sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0\ ,\\
&j = 1, \dots, n\\
&\quad v_i \ge \varepsilon\ ,\ \text{i}=1,\dots,\text{m}\\
&\quad u_r \ge \varepsilon\ ,\ \text{r}=1,\dots,\text{s}\\
&\quad w_p \ge \varepsilon\ ,\ \text{p}=1,\dots,\text{q}
\end{aligned} \tag{7}$$

Model (7) is a bi-level linear programming problem, in which two objective functions apply the restrictions of stages one and two in the DEA network.

3. Two-stage Network Processes

In this section, by considering four different configurations in two-stage network DEA, a number of models are proposed for finding common weight sets.

3.1. Configuration One

In this configuration, the outputs of stage one are inputs in stage two, as illustrated in Fig. 1.

A) In stage one, X = $(x_1,\ldots,x_m)$ is the input vector with the weight vector V=$(v_1,\ldots,v_m)$, and Z=$(z_1,\ldots,z_q)$ is the output vector with the weight vector W=$(w_1,\ldots, w_q)$. To find a common weight set and rank the DMUs in this stage, we first use the following model: (see Despotis et al. (2016))

$$
\begin{aligned}
E_{csw1}:\quad & \text{Min} \quad \sum_{j=1}^{n}\Big(\sum_{i=1}^{m} x_{ij}\, v_i - m - \sum_{p=1}^{q} z_{pj}\, w_p + n\Big) \\
\text{s.t:}\quad & m-\sum_{i=1}^{m} x_{ij}\, v_i \le 0 \ , \quad j=1,\ldots,n \\
& n-\sum_{p=1}^{q} z_{pj}\, w_p \ge 0 \ , \quad j=1,\ldots,n \qquad\qquad (8) \\
& \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0 \ , \quad j=1,\ldots,n \\
& v_i \ge \varepsilon \ , \quad i=1,\ldots,m \\
& w_p \ge \varepsilon \ , \quad p=1,\ldots,q \\
& m\ , n \ge 0
\end{aligned}
$$

where, $m=\min\limits_{1\le j\le n} \sum_{i=1}^{m} x_{ij}\, v_i$, $n=\max\limits_{1\le j\le n} \sum_{p=1}^{q} z_{pj}\, w_p$, and $\mathcal{E}$ is the smallest positive number.

B) For the purposes of calculating a common weight set in the second network stage, Model (9) is suggested:

$$
\begin{aligned}
E_{csw2}:\quad & \text{Min} \quad \sum_{j=1}^{n}\Big(\sum_{p=1}^{q} z_{pj}\, w_p - M - \sum_{r=1}^{s} y_{rj}\, u_r + N\Big) \\
\text{s.t:}\quad & M-\sum_{p=1}^{q} z_{pj}\, w_p \le 0 \ , \quad j=1,\ldots,n \\
& N-\sum_{r=1}^{s} y_{rj}\, u_r \ge 0 \ , \quad j=1,\ldots,n \\
& \sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0 \ , \quad j=1,\ldots,n \qquad\qquad (9) \\
& u_r \ge \varepsilon \ , \quad r=1,\ldots,s \\
& w_p \ge \varepsilon \ , \quad p=1,\ldots,q \\
& M\ , N \ge 0
\end{aligned}
$$

In Model (9), M and N are defined as follows:

$M=\min\limits_{1\le j\le n} \sum_{p=1}^{q} z_{pj}\, w_p$

$N=\max\limits_{1\le j\le n} \sum_{r=1}^{s} y_{rj}\, u_r$

Similarly, Model (6) denoted by $E_{efficiency2}$ is used for efficiency calculation in the second network stage.

To calculate the overall efficiency of the system, we use the following formula:

$$
\begin{aligned}
E_{overall}:\quad & \text{Min} \quad \sum_{j=1}^{n}\Big(\sum_{i=1}^{m} x_{ij}\, v_i - m - \sum_{p=1}^{q} z_{pj}\, w_p + n\Big) + \sum_{j=1}^{n}\Big(\sum_{p=1}^{q} z_{pj}\, w_p - M - \sum_{r=1}^{s} y_{rj}\, u_r + N\Big) \\
\text{s.t:}\quad & m-\sum_{i=1}^{m} x_{ij}\, v_i \le 0 \ , \quad j=1,\ldots,n \\
& n-\sum_{p=1}^{q} z_{pj}\, w_p \ge 0 \ , \quad j=1,\ldots,n \\
& M-\sum_{p=1}^{q} z_{pj}\, w_p \le 0 \ , \quad j=1,\ldots,n \qquad\qquad (10) \\
& N-\sum_{r=1}^{s} y_{rj}\, u_r \ge 0 \ , \quad j=1,\ldots,n \\
& \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0 \ , \quad j=1,\ldots,n \\
& \sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0 \ , \quad j=1,\ldots,n \\
& v_i \ge \varepsilon \ , \quad i=1,\ldots,m \\
& u_r \ge \varepsilon \ , \quad r=1,\ldots,s \\
& w_p \ge \varepsilon \ , \quad p=1,\ldots,q \\
& m\ , n,\ M,\ N \ge 0
\end{aligned}
$$

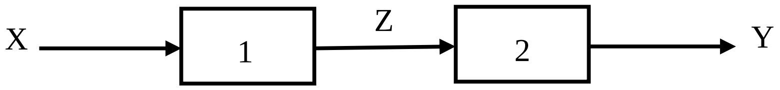

Figure 1. Two-stage network configuration one

3.2. Configuration Two

In this case, an external input enters the system in stage two, as can be seen in Fig. 2:

A) In stage one, the inputs and outputs are similar to the previous configuration, and thus, the formulas (8) and (4) denoted by e_{csw1} and $e_{efficiency2}$, respectively, are successively used for the purposes of unit ranking.

B) In the second stage, we have the input vectors Z=$(z_1,\dots,z_q)$ and L=$(l_1,\dots,l_a)$ with the weight vectors W=$(w_1,\dots,w_q)$ and H=$(h_1,\dots,h_a)$, respectively, and the output vector Y = $(y_1,\dots,y_s)$ with the weight vector U=$(u_1,\dots,u_s)$. For efficiency calculation with common weights, Model (11) is first employed:

e_{csw2}: Min $\sum_{j=1}^{n}(\sum_{p=1}^{q} z_{pj}\, w_p - M_1 + \sum_{d=1}^{a} l_{dj}h_d - M_2 - \sum_{r=1}^{s} y_{rj}\, u_r + N)$

s.t: $M_1 - \sum_{p=1}^{q} z_{pj}\, w_p \le 0$, j=1,…,n

$M_2 - \sum_{d=1}^{a} l_{dj}h_d \le 0$, j=1,…,n

$N - \sum_{r=1}^{s} y_{rj}\, u_r \ge 0$, j=1,…,n (11)

$\sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{d=1}^{a} l_{dj}h_d \le 0$, j=1,…,n

$u_r \ge \varepsilon$, r=1,…,s

$w_p \ge \varepsilon$, p=1,…,q

$h_d \ge \varepsilon$, d=1,…,a

M_1 , M_2, N ≥ 0

In this model,

$M_1 = \min_{1\le j\le n} \sum_{p=1}^{q} z_{pj}\, w_p$,

$M_2 = \min_{1\le j\le n} \sum_{d=1}^{a} l_{dj}h_d$, and

$N = \max_{1\le j\le n} \sum_{r=1}^{s} y_{rj}\, u_r$.

Then, the following model is presented in accordance with Model (6) for efficiency calculation in this case:

$e_{efficiency2}$: Max $\sum_{r=1}^{s} y_{ro}\, u_r$

s.t: $\sum_{p=1}^{q} z_{po}\, w_p + \sum_{d=1}^{a} l_{do}h_d = 1$

$\sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0$, j=1,…,n (12)

$\sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{d=1}^{a} l_{dj}h_d \le 0$, j=1,…,n

$v_i \ge \varepsilon$, i=1,…,m

$u_r \ge \varepsilon$, r=1,…,s

$w_p \ge \varepsilon$, p=1,…,q

$h_d \ge \varepsilon$, d=1,…,a

The following linear programming model is proposed for calculating a common weight set in the overall system within the framework of two-stage network DEA.

$e_{efficiency2}$: Max $\sum_{r=1}^{s} y_{ro}\, u_r$

s.t: $\sum_{p=1}^{q} z_{po}\, w_p + \sum_{d=1}^{a} l_{do}h_d = 1$

$\sum_{p=1}^{q} z_{pj}\, w_p - \sum_{i=1}^{m} x_{ij}\, v_i \le 0$ j=1,…,n

$\sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{d=1}^{a} l_{dj}h_d \le 0$, j=1,…,n (13)

$v_i \ge \varepsilon$, i=1,…,m

$u_r \ge \varepsilon$, r=1,…,s

$w_p \ge \varepsilon$, p=1,…,q

$h_d \ge \varepsilon$, d=1,…,a

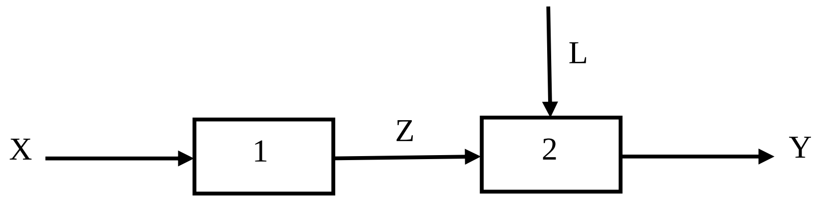

Figure 2. Two-stage network configuration two

3.3. Configuration Three

In this configuration, there is one external output exiting the system in stage one, as demonstrated in Fig. 3.

Consider the example of a university branch with a postgraduate program in which the students first pass an educational phase (stage one) and then go through a research-oriented phase (stage two). In this case, the external output of the education-oriented phase could be students who are transferred to other branches or drop out.

A) In the first stage, X = $(x_1,\dots,x_m)$ is the input vector with the weight vector V=$(v_1,\dots,v_m)$, and Z=$(z_1,\dots,z_q)$ and K=$(k_1,\dots,k_b)$ are output vectors with the weight vectors U=$(u_1,\dots,u_s)$ and T=$(t_1,\dots,t_b)$, respectively. First, Model (14) is proposed as follows for efficiency calculation through common weights:

$$\begin{aligned}
H_{csw1}:\quad & \text{Min} \ \sum_{j=1}^{n}\Big(\sum_{i=1}^{m} x_{ij}\, v_i - m - \sum_{p=1}^{q} z_{pj}\, w_p + n_1 - \sum_{c=1}^{b} k_{cj}\, t_c + n_2\Big)\\
\text{s.t:}\quad & m - \sum_{i=1}^{m} x_{ij}\, v_i \le 0 \ , \quad j=1,\dots,n\\
& n_1 - \sum_{p=1}^{q} z_{pj}\, w_p \ge 0 \ , \quad j=1,\dots,n\\
& n_2 - \sum_{c=1}^{b} k_{cj}\, t_c \ge 0 \ , \quad j=1,\dots,n\\
& \sum_{p=1}^{q} z_{pj}\, w_p + \sum_{c=1}^{b} k_{cj}\, t_c - \sum_{i=1}^{m} x_{ij}\, v_i \le 0 \ , \quad j=1,\dots,n\\
& v_i \ge \varepsilon \ , \ i=1,\dots,m\\
& w_p \ge \varepsilon \ , \ p=1,\dots,q\\
& t_c \ge \varepsilon \ , \ c=1,\dots,b\\
& m \, , n_1, n_2 \ge 0
\end{aligned} \qquad (14)$$

In this model,

$$m = \min_{1\le j\le n} \sum_{i=1}^{m} x_{ij}\, v_i,$$

$$n_1 = \max_{1\le j\le n} \sum_{p=1}^{q} z_{pj}\, w_p, \quad \text{and}$$

$$n_2 = \max_{1\le j\le n} \sum_{c=1}^{b} k_{cj}\, t_c.$$

Next, similar to the previous cases, Model (15) is presented for efficiency calculation in the first stage.

B) To calculate efficiency in stage two, we first use Model (9) represented by H_{csw2}, and then apply Model (6) denoted by $H_{efficiency2}$. For calculating the overall system efficiency using common weights, the following model is suggested:

$$\begin{aligned}
H_{overall}:\quad & \text{Min} \ \sum_{j=1}^{n}\Big(\sum_{i=1}^{m} x_{ij}\, v_i - m - \sum_{p=1}^{q} z_{pj}\, w_p + n_1 - \sum_{c=1}^{b} k_{cj}\, t_c + n_2\Big) + \sum_{j=1}^{n}\Big(\sum_{p=1}^{q} z_{pj}\, w_p - M - \sum_{r=1}^{s} y_{rj}\, u_r + N\Big)\\
\text{s.t:}\quad & m - \sum_{i=1}^{m} x_{ij}\, v_i \le 0 \ , \quad j=1,\dots,n\\
& n_1 - \sum_{p=1}^{q} z_{pj}\, w_p \ge 0 \ , \quad j=1,\dots,n\\
& n_2 - \sum_{c=1}^{b} k_{cj}\, t_c \ge 0 \ , \quad j=1,\dots,n\\
& M - \sum_{p=1}^{q} z_{pj}\, w_p \le 0 \ , \quad j=1,\dots,n\\
& N - \sum_{r=1}^{s} y_{rj}\, u_r \ge 0 \ , \quad j=1,\dots,n\\
& \sum_{p=1}^{q} z_{pj}\, w_p + \sum_{c=1}^{b} k_{cj}\, t_c - \sum_{i=1}^{m} x_{ij}\, v_i \le 0 \ , \quad j=1,\dots,n\\
& \sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p \le 0 \ , \quad j=1,\dots,n\\
& v_i \ge \varepsilon \ , \ i=1,\dots,m\\
& u_r \ge \varepsilon \ , \ r=1,\dots,s\\
& w_p \ge \varepsilon \ , \ p=1,\dots,q\\
& t_c \ge \varepsilon \ , \ c=1,\dots,b\\
& m \, , n_1, n_2, M, N \ge 0
\end{aligned} \qquad (15)$$

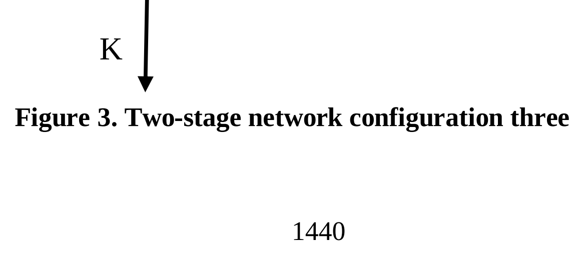

Figure 3. Two-stage network configuration three

3.4. Configuration Four

In this case, an output exits the system in stage one and an input enter the network in stage two; refer to the following figure4:

A) The inputs and outputs in stage one are similar to the case of the third configuration. Therefore, we first make use of Model (14) denoted by h_{csw1}, and then apply Model (15) under the title of $h_{efficiency1}$.

B) Similarly, in stage two, we successively use Models (11) and (12) represented by h_{csw2} and $h_{efficiency2}$, respectively. The following model is proposed for calculating a common weight set in the overall system:

$h_{overall}$: Min $\sum_{j=1}^{n}(\sum_{i=1}^{m} x_{ij}\, v_i - \text{m} - \sum_{p=1}^{q} z_{pj}\, w_p + n_1 - \sum_{c=1}^{b} k_{cj}\, t_c + n_2) + \sum_{j=1}^{n}(\sum_{p=1}^{q} z_{pj}\, w_p - M_1 + \sum_{d=1}^{a} l_{dj} h_d - M_2 - \sum_{r=1}^{s} y_{rj}\, u_r + N)$

s.t $\text{m} - \sum_{i=1}^{m} x_{ij}\, v_i \le 0$, j=1,…,n

$n_1 - \sum_{p=1}^{q} z_{pj}\, w_p \ge 0$, j=1,…,n

$n_2 - \sum_{c=1}^{b} k_{cj}\, t_c \ge 0$,

$M_1 - \sum_{p=1}^{q} z_{pj}\, w_p \le 0$, j=1,…,n (16)

$M_2 - \sum_{d=1}^{a} l_{dj} h_d \le 0$, j=1,…,n

$N - \sum_{r=1}^{s} y_{rj}\, u_r \ge 0$, j=1,…,n

$\sum_{p=1}^{q} z_{pj}\, w_p + \sum_{c=1}^{b} k_{cj}\, t_c - \sum_{i=1}^{m} x_{ij}\, v_i \le 0$, j=1,…,n

$\sum_{r=1}^{s} y_{rj}\, u_r - \sum_{p=1}^{q} z_{pj}\, w_p - \sum_{d=1}^{a} l_{dj} h_d \le 0$, j=1,…,n

$v_i \ge \varepsilon$, i=1,…,m

$u_r \ge \varepsilon$, r=1,…,s

$w_p \ge \varepsilon$, p=1,…,q

$t_c \ge \varepsilon$, c=1,…,b

$h_d \ge \varepsilon$, d=1,…,a

$\text{m}, n_1, n_2, M_1, M_2, N \ge 0$

4. Numerical Example

Example 1: In this section, 40 university branches are considered as two-stage networks, each having two inputs and two outputs in the first stage and two inputs and three outputs in the second stage along with one external input and two external outputs as follows: (figure 5)

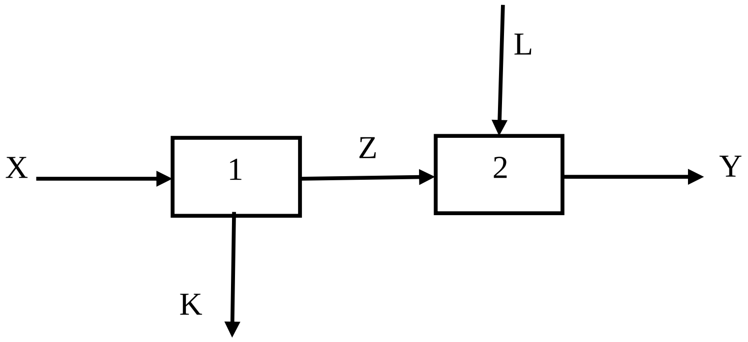

Figure 4. Two-stage network configuration four

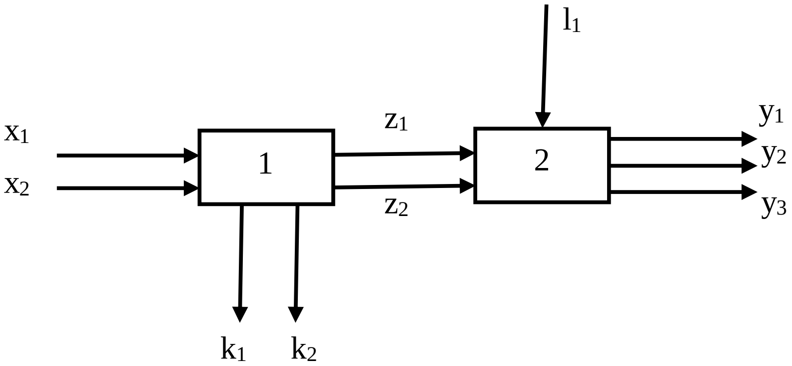

Figure 5. Inputs and outputs in stage one

Table 1 provides the data on the 40 university branches under study. In this example, the inputs and outputs are defined as follows:

Stage one: Education-oriented phase of the postgraduate program

Stage Two: Research-oriented phase of the postgraduate program

X_1: Number of admissions with an entrance exam.

X_2: Number of admissions without an entrance exam

Z_1: Number of graduates from the education-oriented phase with an average of A

Z_2: Number of graduates from the education-oriented phase with an average of B

Y_1: Number of graduates from the research-oriented phase without a research article

Y_2: Number of graduates from the research-oriented phase defending their research article

Y_3: Number of graduates from the research-oriented phase with a research article published in an ISI-indexed journal

L_1: Number of guest students in the research-oriented phase

K_1: Number of expelled students

K_2: Number of drop-out students

Efficiency evaluations were performed for the first two-stage network configuration using Models (4), (6), (8), (9), and (10), the results of which can be observed in Table 2.

In Table 2, by comparing E_{csw1} and $E_{efficiency1}$ in the education-oriented phase, it can be observed that only one university branch is efficient in E_{csw1}, while $E_{efficiency1}$ has deemed 6 branches as efficient. Furthermore, in the research-oriented stage, E_{csw2} presents one efficient unit, while 13 units are efficient according to $E_{efficiency2}$. In other words, in a comparison of units under similar conditions, $E_{efficiency}$ would consider a smaller number of units as efficient comparing to E_{csw}, as $E_{efficiency}$ evaluates the units based on their optimal condition, while E_{csw} determines efficiency scores for unit ranking under the same condition.

Referring to Table 2, it can be said that Branch 5 has done very well in the education-oriented phase, but was not efficient in the research-oriented stage. Moreover, the branch is overall inefficient according to E_{csw1^*} E_{csw2}.

Unit 9 was not efficient in any of the stages or in the overall evaluation.

Unit 13 has performed well in the education-oriented phase based on $E_{efficiency1}$, and is considered efficient, but the unit is not efficient in the research-oriented phase or in the overall evaluation.

Unit 22 is inefficient in the education-oriented stage, but it had an efficient performance in the research-oriented phase; however, the unit is not overall efficient.

Unit 37 is not efficient in the education-oriented stage, but had performed efficiently in the research-oriented phase. The unit is considered overall efficient based on the column E_{csw1^*} E_{csw2}.

Efficiency scores were calculated for the second two-stage network configuration using Models (4), (8), (11), (12), and (13). The results are provided in Table 3.

In Table 3, e_{csw1} has one efficient unit in the education-oriented phase, while $e_{efficiency1}$ considers 6 units as efficient in this stage. In the research-oriented phase, e_{csw2} deems one unit as efficient and $e_{efficiency2}$ has 24 efficient units.

According to Table 3, Unit 5 is efficient in the education-oriented stage, while being efficient in the research-oriented phase. The unit is, however, efficient based on the overall evaluation.

Unit 8 is inefficient in the education-oriented phase and efficient in the research-oriented phase, and it is overall inefficient.

Unit 13 is efficient in both education- and research-oriented stages according to $e_{efficiency1}$ and $e_{efficiency2}$, respectively; however, the unit is deemed overall inefficient.

Unit 19 is not efficient in the first stage, but it is efficient in the research-oriented phase based on $e_{efficiency2}$. Nevertheless, the unit is overall inefficient.

In the following, Figures 8 and 9 illustrate comparisons between the two methods in stages one and two, respectively.

Table 4 presents the efficiency scores calculated for the third network configuration using Models (6), (9), (14), (15), and (16).

In Table 4, 2 units are efficient based on H_{csw1} and 28 units are efficient based on $H_{efficiency1}$ in the education-oriented phase, while in the research-oriented stage, one unit is efficient according to H_{csw1} and 13 units are efficient in $H_{efficiency2}$.

Based on Table 4, Unit 4 is efficient in the education-oriented stage, while it is considered inefficient in the research-oriented phase and the overall evaluation.

Unit 9 is efficient in the first stage, but it is not efficient in the second stage, neither is it overall efficient.

Unit 26 is efficient in the education-oriented phase based on $e_{efficiency1}$. The unit is also efficient in stage two and the overall evaluation.

Unit 38 is efficient in the education-oriented stage according to $e_{efficiency1}$, but it is neither efficient in the research-oriented stage nor the overall evaluation.

The efficiency values measured for the fourth configuration through Models (11), (12), (14), (15), and (17) can be observed in Table 5.

Unit 9 is efficient in the education-oriented phase but inefficient in the research-oriented stage and the overall evaluation.

Unit 29 is efficient in both education- and research-oriented stages according to $h_{efficiency1}$ and $h_{efficiency2}$, respectively. The unit is also overall efficient based on h_{csw1^*} h_{csw2}.

5. Conclusion

In this research, adopting a common perspective on the evaluation of decision-making units, a number of models for determining common weights in data envelopment analysis were explored. Overall, our conclusions fall into two categories:

A. The DEA model for determining common weight sets is a multi-objective fractional programming problem, which is solved using the 1-norm or the infinity norm. In the current research, using the 1-norm and the infinity norm, common weight sets were determined in DEA, and a number of DMUs were ranked on that basis.

B. A number of two-stage network DEA models were proposed in various configurations based on common weight sets. These models are solved in order to rank DMUs considered as two-stage networks.

Generally, finding common weight sets in two-stage network DEA is of great significance. Note that all units are evaluated and ranked in both network stages under similar conditions.

References

[1] Castelli L. Pesenti R. Ukovich W. 2010.A classification of DEA models when the internal structure of the Decision Making Units is considered. Annals of Operations Reasearch, 173: 207-235.

[2] Cook WD. Liang L. Zhu J. 2010. Measuring performance of two-stage network structures by DEA: a review and future perspective. Omega, 38: 423-430.

[3] Despotis D. K. Koronakas G. Sotiros D. 2014. Composition versus decomposition in two-stage network DEA:a reverse approach. Jornal of Productivity analysis, 123:414-415

[4] Despotis D. K. Sotiros D. Koronakos G. 2016. A network DEA approach for series multi-stage processes. Omega, 61: 35-48.

[5] Fare R. Grosskopf S. 1996. Productivity and intermediate products: a frontier approach. Economic Letters, 50: 65-70.

[6] Kao C. 2014. Efficiency decomposition for general multi-stage systems in data envelopment analysis. European Journal of Operational Reasearch, 232: 117-124.

[7] Kao H. Y. Chan C. Y. Wu D. J. 2014.A multi-objective programming method for solving network DEA. Applied Soft Computing, 24: 406-413.

[8] Liang L. Cook WD. Zhu J. 2008. DEA models for two-stage processes: game approach and efficiency decomposition. Naval Reasearch Logistics, 55: 643-653.

Table 1. Data on the 40 university branches under study

DMU	Inputs		External Outputs		Outputs of Stage One		External Input	Outputs		
	X_1	X_2	K_1	K_2	Z_1	Z_2	L_1	Y_1	Y_2	Y_3
1	63	97	12	20	24	51	49	56	37	19
2	119	96	19	59	54	83	26	73	49	9
3	74	68	16	40	30	29	11	38	21	11
4	78	17	17	19	28	31	63	47	36	22
5	147	46	21	36	45	91	59	81	62	38
6	119	52	23	41	56	47	24	29	35	41
7	99	92	24	67	61	33	72	34	67	65
8	124	85	31	43	94	28	55	48	105	54
9	149	95	58	94	38	54	77	68	53	42
10	83	91	32	29	41	62	36	24	69	46
11	93	42	15	46	22	39	63	23	48	32
12	48	99	51	24	14	43	15	32	19	21
13	52	66	34	15	26	43	13	28	32	18
14	89	78	29	38	63	31	42	36	48	25
15	64	88	45	23	39	43	65	17	94	20
16	48	109	19	43	52	24	91	14	53	73
17	54	86	40	22	36	38	52	26	49	38
18	67	99	32	47	26	53	66	48	15	63
19	91	87	41	18	63	38	73	36	57	61
20	31	110	27	32	46	36	33	42	19	39
21	56	68	33	27	42	20	52	62	14	28
22	57	74	31	42	30	22	72	24	70	16
23	48	83	22	31	43	27	36	33	41	32
24	72	37	16	29	38	21	57	24	55	37
25	24	56	20	15	31	13	40	22	41	20
26	82	37	31	46	19	15	68	39	26	26
27	47	52	13	24	32	28	40	22	48	30
28	35	62	17	32	24	24	50	28	29	31
29	31	46	22	16	24	15	63	32	41	29
30	29	62	31	20	19	18	49	26	20	32
31	28	53	21	11	22	24	46	21	33	30
32	32	46	18	19	21	20	68	19	43	37
33	43	64	27	32	22	26	38	16	32	32
34	44	57	21	20	31	24	55	40	20	38
35	89	90	49	30	55	45	63	53	60	35
36	48	38	16	18	26	24	50	30	42	20
37	31	52	13	20	33	16	92	81	21	30
38	98	90	18	23	100	39	39	78	45	35
39	100	68	32	36	55	30	35	60	25	24
40	110	87	36	41	58	37	42	37	46	48

Table 2. Two-stage network configuration one

DMU	Stage One				Stage Two				Overall			
	E_{csw1}		$E_{efficiency1}$		E_{csw2}		$E_{efficiency2}$		$E_{csw1*}\ E_{csw2}$		$E_{overall}$	
1	0.6045	(32)	0.9341	(12)	0.7568	(9)	1.0000	(1)	0.4575	(14)	0.3307	(31)
2	0.8502	(4)	0.9833	(7)	0.4227	(37)	0.6152	(32)	0.3595	(30)	0.2871	(38)
3	0.5438	(35)	0.6168	(37)	0.4627	(35)	0.5742	(36)	0.2516	(40)	0.2380	(39)
4	0.8841	(3)	1.0000	(1)	0.7215	(13)	0.8471	(18)	0.6379	(3)	0.5522	(4)
5	1.0000	(1)	1.0000	(1)	0.6639	(18)	0.8538	(17)	0.6639	(2)	0.4650	(11)
6	0.8230	(5)	0.8820	(16)	0.3880	(38)	0.4546	(40)	0.3193	(36)	0.3089	(36)
7	0.6352	(26)	0.6884	(33)	0.5872	(24)	0.8127	(21)	0.3730	(26)	0.4203	(17)
8	0.7594	(9)	0.9470	(11)	0.4798	(33)	1.0000	(1)	0.3644	(28)	0.4643	(12)
9	0.5098	(36)	0.5565	(39)	0.7886	(8)	0.9025	(16)	0.4020	(19)	0.3280	(32)
10	0.7741	(7)	0.9642	(8)	0.5937	(23)	0.7758	(24)	0.4596	(13)	0.3731	(25)
11	0.6269	(29)	0.6557	(36)	0.7893	(7)	1.0000	(1)	0.4948	(9)	0.3679	(28)
12	0.4952	(39)	0.9497	(10)	0.7505	(10)	1.0000	(1)	0.3716	(27)	0.2364	(40)
13	0.7591	(10)	1.0000	(1)	0.5179	(31)	0.6610	(30)	0.3931	(21)	0.3102	(35)
14	0.7277	(13)	0.7740	(28)	0.3667	(39)	0.5335	(37)	0.2668	(39)	0.3075	(37)
15	0.6904	(19)	0.8563	(17)	0.5839	(25)	1.0000	(1)	0.4031	(18)	0.3694	(26)
16	0.5902	(33)	0.8495	(19)	0.6028	(22)	1.0000	(1)	0.3558	(31)	0.4288	(14)
17	0.6698	(20)	0.8516	(18)	0.6052	(21)	0.6777	(29)	0.4054	(17)	0.3787	(24)
18	0.6148	(31)	0.9225	(13)	0.8678	(5)	1.0000	(1)	0.5335	(6)	0.3912	(21)
19	0.7328	(11)	0.8056	(26)	0.5254	(29)	0.6881	(28)	0.3850	(24)	0.4216	(16)
20	0.7027	(17)	1.0000	(1)	0.4702	(34)	0.5792	(35)	0.3304	(35)	0.3475	(30)
21	0.6327	(27)	0.7151	(31)	0.5688	(27)	0.6912	(27)	0.3599	(29)	0.4226	(15)
22	0.5050	(38)	0.5989	(38)	0.6974	(14)	1.0000	(1)	0.3522	(32)	0.3680	(27)
23	0.6658	(24)	0.8238	(25)	0.5195	(30)	0.5948	(33)	0.3459	(33)	0.3816	(23)
24	0.7256	(14)	0.8267	(21)	0.6404	(19)	0.9160	(15)	0.4647	(12)	0.5108	(5)
25	0.6684	(22)	1.0000	(1)	0.5692	(26)	0.9601	(14)	0.3805	(25)	0.4693	(10)
26	0.3891	(40)	0.4184	(40)	1.0000	(1)	1.0000	(1)	0.3891	(23)	0.3841	(22)
27	0.7820	(6)	0.9122	(15)	0.6134	(20)	0.7163	(26)	0.4797	(10)	0.4718	(9)
28	0.6224	(30)	0.8039	(27)	0.7337	(12)	0.7757	(25)	0.4567	(15)	0.4359	(13)
29	0.6368	(25)	0.7649	(29)	0.8910	(4)	1.0000	(1)	0.5674	(5)	0.6239	(2)
30	0.5055	(37)	0.6727	(34)	0.8494	(6)	1.0000	(1)	0.4294	(16)	0.4196	(18)
31	0.7131	(16)	0.9514	(9)	0.7405	(11)	0.8027	(22)	0.5281	(7)	0.4902	(7)
32	0.6686	(21)	0.8265	(22)	0.9304	(2)	1.0000	(1)	0.6221	(4)	0.5999	(3)
33	0.5723	(34)	0.7314	(30)	0.6960	(15)	0.7860	(23)	0.3983	(20)	0.3570	(29)
34	0.6937	(18)	0.8261	(23)	0.6824	(17)	0.8380	(20)	0.4734	(11)	0.4865	(8)
35	0.7239	(15)	0.8250	(24)	0.5398	(28)	0.6192	(31)	0.3908	(22)	0.3915	(20)
36	0.7675	(8)	0.8428	(20)	0.6856	(16)	0.8405	(19)	0.5262	(8)	0.5029	(6)
37	0.7327	(12)	0.9186	(14)	0.9079	(3)	1.0000	(1)	0.6652	(1)	0.7855	(1)
38	0.9485	(2)	1.0000	(1)	0.3548	(40)	0.5222	(38)	0.3365	(34)	0.4103	(19)
39	0.6664	(23)	0.7068	(32)	0.4367	(36)	0.4941	(39)	0.2910	(38)	0.3239	(34)
40	0.6312	(28)	0.6722	(35)	0.4827	(32)	0.5940	(34)	0.3047	(37)	0.3264	(33)

Table 3. Two-stage network configuration two

DMU	Stage One		Stage Two		Overall	
	e_{csw1}	$e_{efficiency1}$	e_{csw2}	$e_{efficiency2}$	e_{csw1*} e_{csw2}	$e_{overall}$
1	0.6045 (32)	0.9341 (12)	0.8339 (25)	1.0000 (1)	0.5041 (31)	0.9849 (34)
2	0.8502 (4)	0.9833 (7)	0.6851 (40)	1.0000 (1)	0.5825 (21)	0.8731 (38)
3	0.5438 (35)	0.6168 (37)	0.8700 (19)	1.0000 (1)	0.4731 (35)	0.7207 (39)
4	0.8841 (3)	1.0000 (1)	0.8134 (32)	0.9167 (36)	0.7191 (4)	1.7442 (4)
5	1.0000 (1)	1.0000 (1)	0.8402 (24)	0.9821 (29)	0.8402 (1)	1.4559 (8)
6	0.8230 (5)	0.8820 (16)	0.7367 (38)	1.0000 (1)	0.6063 (14)	0.9579 (35)
7	0.6352 (26)	0.6884 (33)	0.9246 (4)	1.0000 (1)	0.5873 (19)	1.2723 (15)
8	0.7594 (9)	0.9470 (11)	1.0000 (1)	1.0000 (1)	0.7594 (2)	1.4195 (11)
9	0.5098 (36)	0.5565 (39)	0.9095 (7)	0.9861 (28)	0.4637 (36)	1.0053 (31)
10	0.7741 (7)	0.9642 (8)	0.8721 (17)	1.0000 (1)	0.6751 (7)	1.1232 (26)
11	0.6269 (29)	0.6557 (36)	0.7729 (36)	1.0000 (1)	0.4845 (33)	1.1400 (23)
12	0.4952 (39)	0.9497 (10)	0.9166 (5)	1.0000 (1)	0.4539 (38)	0.6983 (40)
13	0.7591 (10)	1.0000 (1)	0.8199 (29)	1.0000 (1)	0.6224 (10)	0.9296 (37)
14	0.7277 (13)	0.7740 (28)	0.6977 (39)	0.7338 (40)	0.5077 (30)	0.9327 (36)
15	0.6904 (19)	0.8563 (17)	0.7466 (37)	1.0000 (1)	0.5155 (29)	1.1040 (27)
16	0.5902 (33)	0.8495 (19)	0.8133 ((33)	1.0000 (1)	0.4800 (34)	1.2634 (17)
17	0.6698 (20)	0.8516 (18)	0.8200 (28)	0.9028 (38)	0.5492 (25)	1.1269 (25)
18	0.6148 (31)	0.9225 (13)	0.8859 (13)	1.0000 (1)	0.5447 (27)	1.1665 (22)
19	0.7328 (11)	0.8056 (26)	0.8234 (27)	0.9045 (37)	0.6034 (15)	1.2751 (14)
20	0.7027 (17)	1.0000 (1)	0.8189 (30)	1.0000 (1)	0.5754 (22)	1.0132 (30)
21	0.6327 (27)	0.7151 (31)	0.8901 (12)	1.0000 (1)	0.5632 (23)	1.2681 (16)
22	0.5050 (38)	0.5989 (38)	0.7821 (34)	1.0000 (1)	0.3950 (39)	1.1019 (28)
23	0.6658 (24)	0.8238 (25)	0.8990 (9)	0.9383 (34)	0.5986 (16)	1.1326 (24)
24	0.7256 (14)	0.8267 (21)	0.9148 (6)	0.9908 (27)	0.6638 (8)	1.5764 (5)
25	0.6684 (22)	1.0000 (1)	0.8853 (14)	0.9601 (31)	0.5917 (18)	1.3815 (12)
26	0.3891 (40)	0.4184 (40)	0.8932 (11)	1.0000 (1)	0.3475 (40)	1.1901 (20)
27	0.7820 (6)	0.9122 (14)	0.8953 (10)	0.9781 (30)	0.7001 (5)	1.4200 (10)
28	0.6224 (30)	0.8039 (27)	0.8632 (22)	0.9264 (35)	0.5373 (28)	1.2931 (13)
29	0.6368 (25)	0.7649 (29)	0.9593 (2)	1.0000 (1)	0.6109 (13)	1.8604 (2)
30	0.5055 (37)	0.6727 (34)	0.9057 (8)	1.0000 (1)	0.4578 (37)	1.2380 (19)
31	0.7131 (16)	0.9514 (9)	0.8636 (21)	0.9558 (32)	0.6158 (11)	1.4513 (9)
32	0.6686 (21)	0.8265 (22)	0.8779 (16)	1.0000 (1)	0.5870 (20)	1.7905 (3)
33	0.5723 (34)	0.7314 (30)	0.8711 (18)	1.0000 (1)	0.4985 (32)	1.0645 (29)
34	0.6937 (18)	0.8261 (23)	0.8808 (15)	0.9954 (25)	0.6110 (12)	1.4568 (7)
35	0.7239 (15)	0.8250 (24)	0.8184 (31)	0.8802 (39)	0.5924 (17)	1.1818 (21)
36	0.7675 (8)	0.8428 (20)	0.8413 (23)	0.9552 (33)	0.6457 (9)	1.5305 (6)
37	0.7327 (12)	0.9114 (15)	0.9523 (3)	1.0000 (1)	0.6978 (6)	2.3336 (1)
38	0.9485 (2)	1.0000 (1)	0.7820 (35)	1.0000 (1)	0.7417 (3)	1.2423 (18)
39	0.6664 (23)	0.7068 (32)	0.8287 (26)	0.9952 (26)	0.5522 (24)	0.9905 (33)
40	0.6312 (28)	0.6722 (35)	0.8666 (20)	1.0000 (1)	0.5470 (26)	0.9932 (32)

Table 4. Two-stage network configuration three

DMU	Stage One				Stage Two				Overall			
	H_{csw1}		$H_{efficiency1}$		H_{csw2}		$H_{efficiency2}$		H_{csw1}* H_{csw2}		$H_{overall}$	
1	0.6033	(40)	0.9341	(38)	0.7568	(9)	1.0000	(1)	0.4566	(30)	0.2801	(34)
2	0.9099	(21)	1.0000	(1)	0.4227	(37)	0.6152	(32)	0.3846	(36)	0.2489	(38)
3	0.7513	(39)	0.8334	(40)	0.4627	(35)	0.5742	(36)	0.3476	(38)	0.2054	(39)
4	1.0000	(1)	1.0000	(1)	0.7215	(13)	0.8471	(18)	0.7215	(8)	0.4997	(4)
5	0.9581	(13)	1.0000	(1)	0.6639	(18)	0.8538	(17)	0.6361	(16)	0.4166	(7)
6	0.9298	(18)	1.0000	(1)	0.3880	(38)	0.4546	(40)	0.3608	(37)	0.2738	(35)
7	0.8878	(30)	1.0000	(1)	0.5872	(24)	0.8127	(21)	0.5213	(26)	0.3626	(15)
8	0.8767	(33)	1.0000	(1)	0.4798	(33)	1.0000	(1)	0.4206	(33)	0.4050	(10)
9	1.0000	(1)	1.0000	(1)	0.7886	(8)	0.9025	(16)	0.7886	(6)	0.2869	(31)
10	0.8997	(27)	0.9663	(34)	0.5937	(23)	0.7758	(24)	0.5342	(24)	0.3199	(26)
11	0.8605	(35)	1.0000	(1)	0.7893	(7)	1.0000	(1)	0.6792	(11)	0.3258	(23)
12	0.9300	(17)	1.0000	(1)	0.7505	(10)	1.0000	(1)	0.6980	(9)	0.1984	(40)
13	0.9989	(3)	1.0000	(1)	0.5179	(31)	0.6610	(30)	0.5173	(27)	0.2646	(37)
14	0.9092	(22)	0.9917	(29)	0.3667	(39)	0.5335	(37)	0.3334	(39)	0.2658	(36)
15	0.9846	(5)	1.0000	(1)	0.5839	(25)	1.0000	(1)	0.5749	(20)	0.3141	(27)
16	0.7792	(38)	1.0000	(1)	0.6028	(22)	1.0000	(1)	0.4697	(28)	0.3589	(17)
17	0.9601	(11)	0.9832	(32)	0.6052	(21)	0.6777	(29)	0.5811	(19)	0.3205	(25)
18	0.9042	(26)	1.0000	(1)	0.8678	(5)	1.0000	(1)	0.7847	(7)	0.3318	(22)
19	0.8821	(32)	0.9880	(30)	0.5254	(29)	0.6881	(28)	0.4635	(29)	0.3633	(14)
20	0.9062	(24)	1.0000	(1)	0.4702	(34)	0.5792	(35)	0.4261	(32)	0.2874	(30)
21	0.9637	(9)	1.0000	(1)	0.5688	(27)	0.6912	(27)	0.5482	(22)	0.3610	(16)
22	0.9262	(19)	1.0000	(1)	0.6974	(14)	1.0000	(1)	0.6459	(14)	0.3136	(28)
23	0.8632	(34)	0.9450	(36)	0.5195	(30)	0.5948	(33)	0.4484	(31)	0.3220	(24)
24	0.9075	(23)	1.0000	(1)	0.6405	(19)	0.9160	(15)	0.5813	(18)	0.4503	(5)
25	0.9322	(16)	1.0000	(1)	0.5692	(26)	0.9601	(14)	0.5306	(25)	0.3924	(12)
26	0.9637	(9)	1.0000	(1)	1.0000	(1)	1.0000	(1)	0.9637	(1)	0.3401	(20)
27	0.8968	(28)	0.9856	(31)	0.6134	(20)	0.7163	(26)	0.5501	(21)	0.4043	(11)
28	0.9226	(20)	1.0000	(1)	0.7337	(12)	0.7757	(25)	0.6769	(12)	0.3676	(13)
29	0.9820	(6)	1.0000	(1)	0.8910	(4)	1.0000	(1)	0.8750	(3)	0.5292	(2)
30	0.9747	(7)	1.0000	(1)	0.8494	(6)	1.0000	(1)	0.8279	(4)	0.3517	(19)
31	0.9330	(15)	1.0000	(1)	0.7405	(11)	0.8027	(22)	0.6909	(10)	0.4125	(9)
32	0.9584	(12)	1.0000	(1)	0.9304	(2)	1.0000	(1)	0.8917	(2)	0.5094	(3)
33	0.9712	(8)	1.0000	(1)	0.6960	(15)	0.7860	(23)	0.6760	(13)	0.3028	(29)
34	0.9060	(25)	0.9518	(35)	0.6824	(17)	0.8380	(20)	0.6183	(17)	0.4146	(8)
35	0.9931	(4)	1.0000	(1)	0.5398	(28)	0.6192	(31)	0.5361	(23)	0.3366	(21)
36	0.9372	(14)	0.9772	(33)	0.6856	(16)	0.8405	(19)	0.6425	(15)	0.4364	(6)
37	0.8956	(29)	1.0000	(1)	0.9079	(3)	1.0000	(1)	0.8131	(5)	0.6635	(1)
38	0.8528	(36)	1.0000	(1)	0.3548	(40)	0.5222	(38)	0.3026	(40)	0.3540	(18)
39	0.8839	(31)	0.9371	(37)	0.4367	(36)	0.4941	(39)	0.3860	(35)	0.2826	(33)
40	0.8436	(37)	0.8858	(39)	0.4827	(32)	0.5940	(34)	0.4072	(34)	0.2832	(32)

Table 5. Two-stage network configuration four

DMU	Stage One		Stage Two		Overall	
	h_{csw1}	$h_{efficiency1}$	h_{csw2}	$h_{efficiency2}$	h_{csw1}* h_{csw2}	$h_{overall}$
1	0.6033 (40)	0.9341 (38)	0.8339 (26)	1.0000 (1)	0.5031 (40)	0.5742 (28)
2	0.9099 (21)	1.0000 (1)	0.6851 (40)	1.0000 (1)	0.6234 (39)	0.4995 (35)
3	0.7513 (39)	0.8334 (40)	0.8700 (20)	1.0000 (1)	0.6536 (36)	0.4139 (36)
4	1.0000 (1)	1.0000 (1)	0.8134 (33)	0.9167 (36)	0.8134 (15)	0.9641 (1)
5	0.9581 (13)	1.0000 (1)	0.8402 (25)	0.9821 (29)	0.8050 (18)	0.8112 (8)
6	0.9298 (18)	1.0000 (1)	0.7367 (1)	1.0000 (1)	0.6850 (33)	0.5384 (33)
7	0.8878 (30)	1.0000 (1)	0.9246 (5)	1.0000 (1)	0.8209 (13)	0.7311 (14)
8	0.8767 (33)	1.0000 (1)	1.0000 (1)	1.0000 (1)	0.8767 (4)	0.8082 (9)
9	1.0000 (1)	1.0000 (1)	0.9095 (8)	0.9861 (28)	0.9095 (2)	0.5711 (29)
10	0.8997 (27)	0.9663 (34)	0.8721 (18)	1.0000 (1)	0.7846 (25)	0.6486 (22)
11	0.8605 (35)	1.0000 (1)	0.7729 (37)	1.0000 (1)	0.6651 (35)	0.6413 (24)
12	0.9300 (17)	1.0000 (1)	0.9166 (6)	1.0000 (1)	0.8524 (8)	0.4102 (37)
13	0.9989 (3)	1.0000 (1)	0.8199 (30)	1.0000 (1)	0.8190 (14)	0.5390 (32)
14	0.9092 (22)	0.9917 (29)	0.6977 (39)	0.7338 (40)	0.6343 (37)	0.5350 (34)
15	0.9846 (5)	1.0000 (1)	0.7466 (38)	1.0000 (1)	0.7351 (28)	0.6416 (23)
16	0.7792 (38)	1.0000 (1)	0.8133 (34)	1.0000 (1)	0.6337 (38)	0.7440 (11)
17	0.9601 (11)	0.9832 (32)	0.8200 (29)	0.9028 (38)	0.7873 (24)	0.6577 (21)
18	0.9042 (26)	1.0000 (1)	0.8859 (14)	1.0000 (1)	0.8010 (20)	0.6794 (18)
19	0.8821 (32)	0.9880 (30)	0.8234 (28)	0.9045 (37)	0.7263 (31)	0.7333 (13)
20	0.9062 (24)	1.0000 (1)	0.8189 (31)	1.0000 (1)	0.7421 (27)	0.6022 (27)
21	0.9637 (9)	1.0000 (1)	0.8901 (13)	1.0000 (1)	0.8578 (6)	0.7344 (12)
22	0.9262 (19)	1.0000 (1)	0.7821 (35)	1.0000 (1)	0.7244 (32)	0.6394 (25)
23	0.8632 (34)	0.9450 (36)	0.8990 (10)	0.9383 (34)	0.7760 (26)	0.6624 (20)
24	0.9075 (23)	1.0000 (1)	0.9148 (7)	0.9908 (27)	0.8302 (11)	0.8900 (2)
25	0.9322 (16)	1.0000 (1)	0.8853 (15)	0.9601 (31)	0.8253 (12)	0.8140 (7)
26	0.9637 (9)	1.0000 (1)	0.8932 (12)	1.0000 (1)	0.8608 (5)	0.6694 (19)
27	0.8968 (28)	0.9856 (31)	0.8953 (11)	0.9781 (30)	0.8029 (19)	0.8201 (6)
28	0.9226 (20)	1.0000 (1)	0.8632 (23)	0.9264 (35)	0.7964 (22)	0.7568 (10)
29	0.9820 (6)	1.0000 (1)	0.9593 (3)	1.0000 (1)	0.9420 (1)	1.0836 (39)
30	0.9747 (7)	1.0000 (1)	0.9057 (9)	1.0000 (1)	0.8828 (3)	0.7280 (15)
31	0.9330 (15)	1.0000 (1)	0.8636 (22)	0.9558 (32)	0.8057 (17)	0.8508 (4)
32	0.9584 (12)	1.0000 (1)	0.8779 (17)	1.0000 (1)	0.8414 (10)	1.0420 (40)
33	0.9712 (8)	1.0000 (1)	0.8711 (19)	1.0000 (1)	0.8460 (9)	0.6201 (26)
34	0.9060 (25)	0.9518 (35)	0.8808 (16)	0.9954 (25)	0.7980 (21)	0.8 453 (5)
35	0.9931 (4)	1.0000 (1)	0.8184 (32)	0.8802 (39)	0.8128 (16)	0.6808 (17)
36	0.9372 (14)	0.9772 (33)	0.8413 (24)	0.9552 (33)	0.7885 (23)	0.8752 (3)
37	0.8956 (29)	1.0000 (1)	0.9523 (4)	1.0000 (1)	0.8529 (7)	1.3638 (38)
38	0.8528 (36)	1.0000 (1)	0.7820 (36)	1.0000 (1)	0.6669 (34)	0.7136 (16)
39	0.8839 (31)	0.9371 (37)	0.8287 (27)	0.9952 (26)	0.7325 (29)	0.5638 (31)
40	0.8436 (37)	0.8858 (39)	0.8666 (21)	1.0000 (1)	0.7311 (30)	0.5679 (30)