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Evaluating the Efficiency of Bank Branches with Random Data

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Abstract

Data Envelopment Analysis (DEA) is a mathematic technique to evaluate the relative efficiency of a group of homogeneous decision making units (DMUs) with multiple inputs and outputs. The efficiency of each unit is measured based on its distance to the production possibility set (PPS). [5] (Barberis, N. & Thaler) In this paper, the BCC model is used in output-oriented. The average return on profit as output and the covariance of profit (risk) are considered as inputs. In the continuation, the median and the mod earned investment as two factors of output to the model presented to provide a better analysis of the types of investment, and finally, let us mention a true example.

Keywords: Data Envelopment Analysis (DEA), Stock Portfolio, Model Orientation, Average Variance, Risk.

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1. Introduction

Portfolio optimization problem is one of the most attractive financial and investment issues. The purpose of optimizing the portfolio is to determine the companies and the amount of stocks which an investor can buy. There are two factors that are important in each decision which are named risk and returns. A return of a financial asset can be estimated by considering the future cash flow predicted for it. Note that this is not an accurate estimate and there is always an uncertainty in the calculation of this return, which is named risk. [4] (Banker, R.D., Charnes), The main goal of more investors is to maximize profit and achieve the highest degree of utility. Therefore, the choice of the correct investment method which has direct effect on the rate of return and desirability of it is too important. In data envelopment analysis, efficiency is an indicator that measures the ability of a decision maker to optimally use inputs to generate outputs. The more an efficient unit can produce more outputs with less inputs, the higher its unit performance. Although we would like to apply models in the real world, DEA models have been dealing with the pairs of positive input and output vectors. All data are assumed to be non-negative, but at least one component of each input and output vector is positive. Therefore, we need the models that are able having both positive and negative inputs-outputs. Moreover, in most of the investment structure, there are outputs that are not desirable in management, in other words, increasing such outputs will reduce the performance of the company (undesirable outputs). In this paper, by combining the concept of undesirable outputs with DEA models, a method to select stock portfolios will be presented for investment. Finally, the proposed method will be implemented and analyzed on the actual data of the companies in the stock exchange or exchange market. [7] (Basso, A. & Funari) In this paper, the BCC model in output-oriented is used to investigate the stock portfolios. After that, we have added the mode and median of investment profit as two constraints outputs in order to better analyze investment types. At the end, we have mentioned to the real example.

2. Preliminaries and notations

There are some DEA models which we noted the BCC model here.

The under evaluated unit, DMUP, is the relative efficient if it was on the efficiency frontier (it means that its efficiency score be equal by one) otherwise it is an inefficient unit (when its efficiency score is less than one). There are some methods, as reducing the inputs or increasing the output, to reach to the efficiency when a DMU is inefficient.

2-1. The BCC model.

This model introduced by Banker, Charness and Cooper [1]. To make this model, the assumption of constant return to scale is removed and the new PPS is created by four principles (observed, free disposability, convexity, minimum extrapolation)

$T_V = \{ (X,Y): X \geq \sum_{j=1}^{n} \lambda_j X_j \;\&\; Y \leq \sum_{j=1}^{n} \lambda_j Y_j \& \sum_{j=1}^{n} \lambda_j = 1 \;\& \lambda_j \geq 0, j = 1, \dots, n \}$

2-1. The BCC model in the input-oriented.

This set is the only one which satisfies in observed, free disposability, convexity, minimum extrapolation. T_v or T_{BCC} is used to represent the production possibility set of the production technology which is satisfy in four principles.

$\min \theta$

$s.t \quad (\theta\, X_o , Y_o) \in \; T_v$

By assignment the T_v condition we have the liner programing as follow:

$$\begin{aligned}
&Min\ \theta \\
&s.t\quad \sum_{j=1}^{n}\lambda_j X_j \le \theta X_o \\
&\sum_{j=1}^{n}\lambda_j Y_j \ge Y_o \\
&\sum_{j=1}^{n}\lambda_j = 1 \\
&\lambda_j \ge 0\ ,j=1,\dots,n
\end{aligned} \tag{1-2}$$

2-2. The BCC model in the output-oriented.

There is the envelopment form of the BCC model in the output-oriented that it obtained as above.

$$\begin{aligned}
&Max\quad \varphi \\
&s.t\quad \sum_{j=1}^{n}\lambda_j X_j \le X_o \\
&\sum_{j=1}^{n}\lambda_j Y_j \ge \varphi Y_o \\
&\sum_{j=1}^{n}\lambda_j = 1 \\
&\lambda_j \ge 0\ ,j=1,\dots,n
\end{aligned} \tag{2-2}$$

In the BCC model, the PPS is created by the convex composition of the observed units also the relative efficiency of this model is obtained based on variable return to scale. Therefore, the efficiency in the BCC model is named with technical localized efficiency. Indeed, if the DMU is efficient in the BCC model it will do well as local but no in global. [9] (Basso, A. & Funari)

3. Innovation: the effect of mode and median on portfolio analysis

Generally, positive conditions are assumed in the production theory for input and output variables while these conditions are useable only within a framework. Because, financial assets often have negative returns (for instance during the recession). Negative data are important not only for negative returns, but also for all mathematical methods of distribution methods that potentially enter a set of output variables. Additionally, if negative values are difficult, especially when using financial performance ratios, DEA provides supplementary items for the use of financial assets to handle negative amounts.

In this paper, the BCC model (output-oriented) is used to analyze portfolio. In this model, risk covariance and average return are considered as input and output, respectively. Also, the portfolio has negative inputs and outputs.

The directional distance function and a positive number, as an improved vector coefficient, are used in this paper when the risk covariance (input) and the average return (output) are negative. The goal is to maximize these positive numbers so that inputs and outputs are transmitted to the efficiency boundary and considered. Because the risks and returns have the same nature then the coefficient τ, defined as $0 \le \tau \le 1$, is used to fit the risk covariance (input) and the average return (output). In fact, any benefit derived from investing is an output and this seems to be consistent with risk taking as an output. On the other hand, no operational form is capable of expressing higher return expectations as a result of investment with a higher risk. Therefore, in this model risk seems to be considered as an input unit and by τ coefficient, the input and outputs are proportioned.

$$\begin{aligned}
&\min\quad \{\delta^2\} \\
&s.t.\quad \tau\sum_{j=1}^{n} q_j\,\mu_j \ge \mu_{jo} \\
&\tau^2\sum_{j=1}^{n}\sum_{k=1}^{n} q_j\,q_k\,\sigma_{jk} \le \delta^2\sigma_{jo}^2 \\
&\sum_{j=1}^{n} q_j = 1 \\
&q_j \ge 0\quad ,for\ all\ j \\
&0 \le \tau \le 1
\end{aligned} \tag{3-2}$$

where in:

q_j: represents the stock of each portfolio in the investment;

q_k: indicates the dispersion matrix of DMUs;

σ_{jo}^2: represents the return variance (risk) of the portfolio;

μ_{jo}: indicates the average return of the portfolio.

This study was carried out over a period of three years and ultimately, this result was obtained:
When the data of a portfolio is multiplied by τ $(0 \leq \tau \leq 1)$, the yield covariance (input) is multiplied by τ^2 and the average yield is multiplied by τ.

3-1. The BCC model in the presence of average, mode, median and covariance:
Definition (Median). The median is a number which divides a statistical population or a possible distribution into two equal parts. If the number of statistical population is even, the median is calculated with the average of two members of the population that are located in the middle of statistical population.

$$N = L + \frac{\frac{N}{2} - f_{ci}}{f_i} i \qquad (1\text{-}3)$$

where in:
L: represents real inferior limit of the stratum that has the highest frequency;
f_{ci}: indicates the cumulative frequency which is the sum of absolute frequencies from the first stratum to the ith stratum that is represented by f_{ci}.
i: represents the length or distance of the stratums.
Definition (Mode). In statistics or mathematics, the value or amount that occurs most frequently in a statistical set is called mode, which is a central tendency measure.
The mode for the continuous probability distribution is X, which is the probability density function of the maximum value. So, the mode is on the peak.
Remark. There is no mode if the repetition of the attribute has occurred equally among statistical data.
where in:
i: represents the length or distance of the stratums.
d_1: indicates the difference between the stratum includes mode and the previous stratum.
d_2: indicates the difference between the stratum includes mode and the next stratum.

$$M = L + i\left(\frac{d_1}{d_1 + d_2}\right) \qquad (2\text{-}3)$$

In this section, we exert median and mode on the BCC model on the presence of average output vector of covariance, and then we consider portfolio.
Consider model 3-2 in which we exert the median and mode output vectors on model (3-1):

$$\begin{aligned}
&max \quad \{\gamma_e^t\} \\
&s.t. \quad \sum_{j=1}^{n} q_j^t \mu_j^t \geq \mu_{jo}^t + \gamma_e^t g_\mu^t \\
&\sum_{j=1}^{n}\sum_{k=1}^{n} q_j^t q_k^t \sigma_{ik}^t \leq \sigma_{jo}^{2^t} - \gamma_e^t g_{\sigma^2}^t \\
&\sum_{j=1}^{n} q_j^t N_j^t \geq N_{jo}^t + \gamma_e^t g_N^t \qquad (3\text{-}3) \\
&\sum_{j=1}^{n} q_j^t m_j^t \geq m_{jo}^t + \gamma_e^t g_m^t \\
&\sum_{j=1}^{J} q_j^t = 1 \\
&q_j^t \geq 0 \quad for\ all\ j
\end{aligned}$$

where we represent mode with m and median with N.
T:time representative
J:represents the number of DMUs examined
Now, we consider the effect of τ coefficient in the above model and conclude that when the data of a portfolio is multiplied by τ $(0 \leq \tau \leq 1)$, the median and mode are multiplied by τ. Therefore, the stock of the company with more median and mode will be more suitable for investment. Finally, model (3-3) is rewritten while multiplied by τ and the inputs and outputs are fitted.

$$\begin{aligned}
&max \quad \{\gamma_e^t\} \\
&s.t. \quad \tau\sum_{j=1}^{n} q_j^t \mu_j^t \geq \mu_{jo}^t + \gamma_e^t g_\mu^t \\
&\tau^2\sum_{j=1}^{n}\sum_{k=1}^{n} q_j^t q_k^t \sigma_{ik}^t \leq \sigma_{jo}^{2^t} - \gamma_e^t g_{\sigma^2}^t \\
&\tau\sum_{j=1}^{n} q_j^t N_j^t \geq N_{jo}^t + \gamma_e^t g_N^t \\
&\tau\sum_{j=1}^{n} q_j^t m_j^t \geq m_{jo}^t + \gamma_e^t g_m^t \qquad (3\text{-}3) \\
&\sum_{j=1}^{J} q_j^t = 1 \\
&q_j^t \geq 0 \text{ for all j and } 0 \leq \tau \leq 1 \ )
\end{aligned}$$

model description:
Model's Input: Generally, as shown in the

first level of Model 3-3, variance is considered as an input, because the less one is better, (variance indicates the investment's risk). In this constraint, compared to the other companies, the variable γ maximizes improvement or reduction of risk. This minimize is shown on the right of this constraint by the subtraction function of the previous value of variance.

Model's Output:

1. We know that one of the main factors of investing which is interested to managers and investors is returns on profit on investment. Therefore, in this paper, returns on profit is considered as an output which is shown in the second level of Model 3-3. In this constraint, by the addition function of the previous amount of investment returns, the variable γ yields the maximum amount of improvement or increase to the right of this constraint.

2. According to the median's definition, (The median is a number that divides a statistical population or a probability distribution into two equal parts. If the population's number is even, the median is obtained by averaging two members of the middle population.) For the median calculation, the number of data is important, while large and small values do not matter. Therefore, over a period of time, a company is better for investing that have a higher median in a few months or years. In this paper, the median of returns on profit is intended as an output which is shown in the third level of Model 3-3. In this constraint, compared to the other companies, the variable γ yields the maximum amount of improvement or increase the median of return on investment profit. This increase is shown on the right-hand side of the constraint by adding function of the previous value the median of returns on profit.

3. According to the mode's definition, (In statistics or math, this is what happens most often). In mode, the type of data is not important. Therefore, mode is used for quantitative and qualitative data. In fact, the mode is the most frequented value in a dataset. A company that has the highest mode of returns of profit is better for investing. In this paper, the mode of returns on profit is intended as an output which is shown in the fourth level of Model 3-3. In this constraint, compared to the other companies, the variable γ yields the maximum amount of improvement or increase the mode of return on investment profit. This increase is shown on the right-hand side of the constraint by adding function of the previous value the mode of returns on profit.

Comparison of the (3-1) and (3-3) models:

Compared to model 3-1 and companies in order to investing model 3-3 has more criteria for companies. By having a median and mode of the investing interval, we can understand the profits of a company for the maximum and minimum period of time. We also determine the duration of a company's downturn. On the other hand, this review can be time consuming for the investor and therefore, comparing with other models is not possible.

4. Numerical examples

In this section, the applied analysis of the method presented in 3-1 is given. For this issue, twenty branches of bank have selected and their information has investigated in a period 30 month.

4-1. Introducing DATA

To evaluating the performance of these branches, the BCC model has utilized in output-oriented. The input variable includes the covariance of these branches, which is used as an investment risk. Also, the average profits of these branches over a period of 30 months, mode and the profit median during this period were used as output

The figures in Table 1 indicate the fact that DMU3 and DMU20 are on the efficiency frontier and investing in these two branches is better than other branches. Moreover, DMU6, DMU7, DMU8 and DMU9 are at the farthest distance to the efficiency frontier so these branches are weak, in terms of performance, compared to the others. Also, the outputs and inputs for all DMUs are in proportional state due to the given number obtained for the coefficient τ, that is equal one to all DMUs.

5. Conclusion.

In this paper, the BCC model, in output-oriented, is used to analyze stock portfoliosThe investment risk is considered as input and the average return on profit is assumed as output in this model. .(Anderson, R .I., Brockman) [2] In order to deal with undesirable inputs and output, we have used the directional distance vector which γ was introduced as the coefficient of this vector. The stock portfolio performance is evaluated by the directional vector versus the efficiency frontier. Besides, in this research, to better analyze the performance of investment firms, we have added the median and investment returns as two output vectors to the model. These results suggest that the more the mode and median to investment, the more suitable company for investment. The coefficient τ is considered to coordinate the investment factors (risk, average return, median, Mode) that is $0 \leq \tau \leq 1$. The result shows that the median and the mode are multiplied by the number τ when the company' profit is multiplied by it. Finally, as an empirical analysis, we have examined the performance of 30 bank branches over a period of 30 months. At the end, we suggest analyzing the returns to scale and sensitivity analysis in the stock portfolio with the presence of these factors (risk, average return, median, mode)

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20 month profitability table (3-1)

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}
t_1	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189
t_2	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439
t_3	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439
t_4	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439
t_5	2180233281	1177481189	2180233281	1177481189	2180233281	1177481189	2180233281	1177481189	2180233281	1177481189
t_6	2827881894	898139716	2827881894	898139716	2827881894	898139716	2827881894	898139716	2827881894	898139716
t_7	1248187471	803714607	803714607	141749859	112980454	90176840	57267382	213334	2149974701	938602896
t_8	3152096902	2215166438	2173044152	104581226	80297587	60161871	30590930	9959923	3589551906	1509727956
t_9	2949686189	2785783484	979134463	432263487	410937293	362689979	18960924	13991246	2954600623	1022092909
t_{10}	429951797	413829553	152140113	140850377	130181119	116288765	63718208	19084811	786608337	358255083
t_{11}	3243086352	1410903274	450863400	285598876	140536975	122884851	118144749	58434889	3688945629	1584184800
t_{12}	548767463	382981207	299253850	30471969	27477120	8369056	8042667	2576366	719689490	338737036
t_{13}	614061631	449164982	440229529	63549207	40366741	10036849	3342819	1212056	620120373	310522316
t_{14}	873851255	873851255	864714577	35473717	35473717	30023717	17503348	13291836	1503226986	691587616
t_{15}	80471253	57513464	36085703	24563232	20120712	13027657	5495252	1463420	164654888	78239046
t_{16}	1419608911	760756995	487913985	325023846	65140662	47983672	33553538	23342743	842609289	510140329
t_{17}	724975055	717158295	716022091	51347047	46761188	17222290	4960961	2934247	1181978558	706609712
t_{18}	349446617	301181090	290123623	212356868	164027618	98927250	67096400	14499250	3766766470	2346494462
t_{19}	858761201	841789391	244882194	133523237	129540361	115840365	97687621	41992027	1006313839	484026802
t_{20}	956486470	868910531	692378125	490825479	407354330	299341374	212146609	59333121	744623773	383683457
t_{21}	699941744	691012429	691012429	32502860	31723463	21651135	21651135	2129170	893831962	379722113
t_{22}	63277349	47950226	47131380	44369735	42988910	18185890	18185890	3342055	625261710	305920250
t_{23}	1362800859	552300055	539527452	207050500	207050500	179974123	23671500	7989310	1917514201	784598059
t_{24}	939368172	242739934	236526235	216573356	214798014	214798014	134655070	9803836	1332095197	699499009
t_{25}	1177798314	1151619824	1187521977	90916400	72048400	44602400	40459900	9804000	2679984210	1314170755
t_{26}	1749357634	1236202708	1062656850	420971871	285794411	183079068	110974495	14078484	8235994038	4634488694
t_{27}	1248187471	803714607	803714607	141749859	112980454	90176840	57267382	213334	2149974701	938602896
t_{28}	3152096902	2215166438	2173044152	104581226	80297587	60161871	30590930	9959923	3589551906	1509727956
t_{29}	2949686189	2785783484	979134463	432263487	410937293	362689979	18960924	13991246	2954600623	1022092909
t_{30}	429951797	413829553	152140113	140850377	130181119	116288765	63718208	19084811	786608337	358255083

	DMU_{11}	DMU_{12}	DMU_{13}	DMU_{14}	DMU_{15}	DMU_{16}	DMU_{17}	DMU_{18}	DMU_{19}	DMU_{20}
t_1	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189	1077805189
t_2	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439	1122336439
t_3	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439	1150631439
t_4	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439	1158981439
t_5	2180233281	1177481189	2180233281	1177481189	2180233281	1177481189	2180233281	1177481189	2180233281	1177481189
t_6	2827881894	898139716	2827881894	898139716	2827881894	898139716	2827881894	898139716	2827881894	898139716
t_7	871615396	837964922	789536703	740333288	682833774	124301900	59035131	43029309	14895529	5
t_8	1436547076	1421141105	1414018418	1229926287	1224560677	318244966	267118908	243302461	201203260	5315969
t_9	1019676241	999703241	955958818	935135234	919109408	337530441	14695913	10308836	10308836	2034075
t_{10}	313183838	313183838	290985278	252228994	209971409	186585548	144493003	121497197	105067655	50213109
t_{11}	1288144991	833835074	719422451	581451429	517100009	378711385	288650550	214872001	139478593	56637463
t_{12}	313586350	313586350	248022952	240542844	27400261	21314782	10499178	8652822	5348712	568767
t_{13}	305788069	289060397	240846940	219203692	191413166	82639980	58522780	39845574	5933131	3664755
t_{14}	705084164	695661242	693294092	670391342	125705646	97291502	91077452	83975925	31550671	20569511
t_{15}	74427074	70998494	54748687	43093682	30681081	10262184	5516299	3226405	23671	23671
t_{16}	503966364	498604861	476980036	456978119	422415695	119274051	119274051	89214051	70179049	3956167
t_{17}	636553200	627116556	616855613	612447404	604939193	16045069	6480000	4689000	2025000	0
t_{18}	2319729852	2284393992	2256382892	2130675856	2130675856	203189557	182329057	162355757	137441707	30181000
t_{19}	475415843	474030910	459772800	456690691	453055567	47796877	38032494	38032494	17574796	887671
t_{20}	312267089	254513673	220375202	200023584	128841220	83509645	58730082	31756971	3996000	1332000
t_{21}	378058340	374865189	370081628	339491501	340152323	27258902	7236986	8075342	527671	527671
t_{22}	285405180	284024358	284024358	293749283	273776673	149025188	122308729	93742397	60122930	11130114
t_{23}	773517473	760943465	680359551	680359551	665712416	25152661	22489647	17163619	10025661	7362261
t_{24}	694066680	646612980	624923113	199091761	155431053	131286398	114910641	108080505	27453723	17161642
t_{25}	1303918721	1277860612	1265867361	1249238361	1245129614	80578014	63229267	71971267	52261767	33265217
t_{26}	4618750536	4576238536	4420339701	4393062537	4393062537	2581170306	2545628958	2432976674	1424394550	691599081
t_{27}	871615396	837964922	789536703	740333288	682833774	124301900	59035131	43029309	14895529	5747741
t_{28}	1436547076	1421141105	1414018418	1229926287	1224560677	318244966	267118908	243302461	201203260	5315969
t_{29}	1019676241	999703241	955958818	935135234	919109408	337530441	14695913	10308836	10308836	2034075
t_{30}	313183838	313183838	290985278	252228994	209971409	186585548	144493003	121497197	105067655	50213109

20 month variance table (3-2)

(σ_{jk}) variance

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}
t_1	1.00182E+18	6.64E+17	3.95E+17	7.28E+17	8.86E+17	2.54E+17	2.72E+17	3.8E+17	5.93E+17	3.2E+17
t_2	1.59361E+18	1.1E+18	6.54E+17	1.23E+18	1.45E+18	4.63E+17	4.76E+17	6.56E+17	6.9E+17	5.4E+17
t_3	1.30404E+18	1.07E+18	6.36E+17	1.2E+18	1.49E+18	4.44E+17	5.18E+17	6.42E+17	1.05E+18	5.4E+17
t_4	3.18719E+17	1.96E+17	1.11E+17	1.95E+17	2.35E+17	6.82E+16	7.08E+16	1.08E+17	2.17E+17	9.25E+16
t_5	1.52198E+18	1.06E+18	7.27E+17	1.35E+18	1.62E+18	4.49E+17	5.11E+17	6.91E+17	4.92E+17	5.07E+17
t_6	3.11497E+17	2.13E+17	1.31E+17	2.39E+17	2.77E+17	8.65E+16	9.22E+16	1.26E+17	1.46E+17	1.03E+17
t_7	2.88837E+17	2.16E+17	1.31E+17	2.45E+17	2.89E+17	8.85E+16	9.69E+16	1.3E+17	1.33E+17	1.05E+17
t_8	6.50381E+17	4.13E+17	2.22E+17	3.94E+17	4.51E+17	1.56E+17	1.52E+17	2.19E+17	4.17E+17	2.02E+17
t_9	7.14121E+16	4.41E+16	2.6E+16	4.63E+16	5.63E+16	1.67E+16	1.74E+16	2.45E+16	4.71E+16	2.14E+16
t_{10}	4.26046E+17	3.72E+17	2.6E+17	5.12E+17	6.03E+17	1.69E+17	2.11E+17	2.51E+17	1.44E+17	1.86E+17
t_{11}	5.70622E+17	3.4E+17	1.74E+17	3.16E+17	3.51E+17	1.28E+17	1.16E+17	1.77E+17	3.38E+17	1.67E+17
t_{12}	1.8433E+18	7.27E+17	3.04E+17	4.73E+17	4.53E+17	1.86E+17	9.45E+16	2.74E+17	1.37E+18	3.38E+17
t_{13}	5.29529E+17	3.68E+17	2.17E+17	4.05E+17	5.02E+17	1.43E+17	1.5E+17	2.25E+17	2.74E+17	1.77E+17
t_{14}	1.62049E+17	2.24E+17	1.53E+17	2.97E+17	3.41E+17	1.07E+17	1.45E+17	1.5E+17	9.45E+16	1.16E+17
t_{15}	3.69315E+17	2.53E+17	1.38E+17	2.6E+17	2.96E+17	1.14E+17	1.07E+17	1.43E+17	1.86E+17	1.28E+17
t_{16}	9.40447E+17	7.49E+17	5.06E+17	9.8E+17	1.48E+18	2.96E+17	3.41E+17	5.02E+17	4.53E+17	3.51E+17
t_{17}	9.59271E+17	6.61E+17	4.24E+17	8.15E+17	9.8E+17	2.6E+17	2.97E+17	4.05E+17	4.73E+17	3.16E+17
t_{18}	5.84452E+17	3.68E+17	2.41E+17	4.24E+17	5.06E+17	1.38E+17	1.53E+17	2.17E+17	3.04E+17	1.74E+17
t_{19}	1.26642E+18	7.13E+17	3.68E+17	6.61E+17	7.49E+17	2.53E+17	2.24E+17	3.68E+17	7.27E+17	3.4E+17
t_{20}	3.63145E+18	1.27E+18	5.84E+17	9.59E+17	9.4E+17	3.69E+17	1.62E+17	5.3E+17	1.84E+18	5.71E+17
t_{21}	1.00182E+18	6.64E+17	3.95E+17	7.28E+17	8.86E+17	2.54E+17	2.72E+17	3.8E+17	5.93E+17	3.2E+17
t_{22}	1.59361E+18	1.1E+18	6.54E+17	1.23E+18	1.45E+18	4.63E+17	4.76E+17	6.56E+17	6.9E+17	5.4E+17
t_{23}	1.30404E+18	1.07E+18	6.36E+17	1.2E+18	1.49E+18	4.44E+17	5.18E+17	6.42E+17	1.05E+18	5.4E+17
t_{24}	3.18719E+17	1.96E+17	1.11E+17	1.95E+17	2.35E+17	6.82E+16	7.08E+16	1.08E+17	2.17E+17	9.25E+16
t_{25}	1.52198E+18	1.06E+18	7.27E+17	1.35E+18	1.62E+18	4.49E+17	5.11E+17	6.91E+17	4.92E+17	5.07E+17
t_{26}	3.11497E+17	2.13E+17	1.31E+17	2.39E+17	2.77E+17	8.65E+16	9.22E+16	1.26E+17	1.46E+17	1.03E+17
t_{27}	2.88837E+17	2.16E+17	1.31E+17	2.45E+17	2.89E+17	8.85E+16	9.69E+16	1.3E+17	1.33E+17	1.05E+17
t_{28}	6.50381E+17	4.13E+17	2.22E+17	3.94E+17	4.51E+17	1.56E+17	1.52E+17	2.19E+17	4.17E+17	2.02E+17
t_{29}	7.14121E+16	4.41E+16	2.6E+16	4.63E+16	5.63E+16	1.67E+16	1.74E+16	2.45E+16	4.71E+16	2.14E+16
t_{30}	4.26046E+17	3.72E+17	2.6E+17	5.12E+17	6.03E+17	1.69E+17	2.11E+17	2.51E+17	1.44E+17	1.86E+17

	DMU_{11}	DMU_{12}	DMU_{13}	DMU_{14}	DMU_{15}	DMU_{16}	DMU_{17}	DMU_{18}	DMU_{19}	DMU_{20}
t_1	4.42E+17	4.68E+16	4.09E+17	2.28E+17	2.23E+17	1.18E+18	2.01809E+17	1.22856E+18	1.14182E+18	7.09032E+17
t_2	8.08E+17	7.18E+16	6.58E+17	4.07E+17	3.95E+17	2.19E+18	2.97886E+17	1.85591E+18	2.09782E+18	1.14182E+18
t_3	7.43E+17	8.66E+16	7.32E+17	3.75E+17	3.61E+17	1.82E+18	3.7563E+17	2.59723E+18	1.85591E+18	1.22856E+18
t_4	1.05E+17	1.4E+16	1.25E+17	6.1E+16	6.03E+16	2.99E+17	6.71434E+16	3.7563E+17	2.97886E+17	2.01809E+17
t_5	9.16E+17	7.25E+16	6.21E+17	4.29E+17	4.23E+17	2.54E+18	2.98548E+17	1.81549E+18	2.18691E+18	1.1824E+18
t_6	1.54E+17	1.42E+16	1.3E+17	7.81E+16	7.75E+16	4.23E+17	6.03499E+16	3.60893E+17	3.95203E+17	2.22989E+17
t_7	1.61E+17	1.42E+16	1.31E+17	8.15E+16	7.81E+16	4.29E+17	6.10082E+16	3.74934E+17	4.07137E+17	2.27623E+17
t_8	2.27E+17	2.8E+16	2.66E+17	1.31E+17	1.3E+17	6.21E+17	1.25088E+17	7.31686E+17	6.58155E+17	4.09313E+17
t_9	2.66E+16	3.31E+15	2.8E+16	1.42E+16	1.42E+16	7.25E+16	1.40315E+16	8.6591E+16	7.18018E+16	4.67893E+16
t_{10}	3.73E+17	2.66E+16	2.27E+17	1.61E+17	1.54E+17	9.16E+17	1.05001E+17	7.42704E+17	8.07844E+17	4.42386E+17
t_{11}	1.86E+17	2.14E+16	2.02E+17	1.05E+17	1.03E+17	5.07E+17	9.24939E+16	5.40266E+17	5.40498E+17	3.20185E+17
t_{12}	1.44E+17	4.71E+16	4.17E+17	1.33E+17	1.46E+17	4.92E+17	2.17071E+17	1.04575E+18	6.8969E+17	5.93281E+17
t_{13}	2.51E+17	2.45E+16	2.19E+17	1.3E+17	1.26E+17	6.91E+17	1.08214E+17	6.4227E+17	6.55823E+17	3.79707E+17
t_{14}	2.11E+17	1.74E+16	1.52E+17	9.69E+16	9.22E+16	5.11E+17	7.08047E+16	5.18421E+17	4.76238E+17	2.72196E+17
t_{15}	1.69E+17	1.67E+16	1.56E+17	8.85E+16	8.65E+16	4.49E+17	6.82412E+16	4.43939E+17	4.62861E+17	2.5438E+17
t_{16}	6.03E+17	5.63E+16	4.51E+17	2.89E+17	2.77E+17	1.62E+18	2.34741E+17	1.48784E+18	1.44941E+18	8.86207E+17
t_{17}	5.12E+17	4.63E+16	3.94E+17	2.45E+17	2.39E+17	1.35E+18	1.94731E+17	1.20175E+18	1.22587E+18	7.27764E+17
t_{18}	2.6E+17	2.6E+16	2.22E+17	1.31E+17	1.31E+17	7.27E+17	1.10876E+17	6.36239E+17	6.54315E+17	3.94721E+17
t_{19}	3.72E+17	4.41E+16	4.13E+17	2.16E+17	2.13E+17	1.06E+18	1.95894E+17	1.06584E+18	1.10163E+18	6.64497E+17
t_{20}	4.26E+17	7.14E+16	6.5E+17	2.89E+17	3.11E+17	1.52E+18	3.18719E+17	1.30404E+18	1.59361E+18	1.00182E+18
t_{21}	4.42E+17	4.68E+16	4.09E+17	2.28E+17	2.23E+17	1.18E+18	2.01809E+17	1.22856E+18	1.14182E+18	7.09032E+17
t_{22}	8.08E+17	7.18E+16	6.58E+17	4.07E+17	3.95E+17	2.19E+18	2.97886E+17	1.85591E+18	2.09782E+18	1.14182E+18
t_{23}	7.43E+17	8.66E+16	7.32E+17	3.75E+17	3.61E+17	1.82E+18	3.7563E+17	2.59723E+18	1.85591E+18	1.22856E+18
t_{24}	1.05E+17	1.4E+16	1.25E+17	6.1E+16	6.03E+16	2.99E+17	6.71434E+16	3.7563E+17	2.97886E+17	2.01809E+17
t_{25}	9.16E+17	7.25E+16	6.21E+17	4.29E+17	4.23E+17	2.54E+18	2.98548E+17	1.81549E+18	2.18691E+18	1.1824E+18
t_{26}	1.54E+17	1.42E+16	1.3E+17	7.81E+16	7.75E+16	4.23E+17	6.03499E+16	3.60893E+17	3.95203E+17	2.22989E+17
t_{27}	1.61E+17	1.42E+16	1.31E+17	8.15E+16	7.81E+16	4.29E+17	6.10082E+16	3.74934E+17	4.07137E+17	2.27623E+17
t_{28}	2.27E+17	2.8E+16	2.66E+17	1.31E+17	1.3E+17	6.21E+17	1.25088E+17	7.31686E+17	6.58155E+17	4.09313E+17
t_{29}	2.66E+16	3.31E+15	2.8E+16	1.42E+16	1.42E+16	7.25E+16	1.40315E+16	8.6591E+16	7.18018E+16	4.67893E+16
t_{30}	3.73E+17	2.66E+16	2.27E+17	1.61E+17	1.54E+17	9.16E+17	1.05001E+17	7.42704E+17	8.07844E+17	4.42386E+17

Table of mean and median return and profit model (3-3)

	Average (μ_j)	**Middle** (N_j)	**method** (M_j)
DMU_1	929636221.333	820839764.500	803714607
DMU_2	1478760080.233	1117908400.000	985006123
DMU_3	1752164421.733	1009689741.000	10308836
DMU_4	361313742.433	313183838.000	313183838
DMU_5	1264312584.067	576763676.000	583078012
DMU_6	273517820.433	241636836.500	229612896
DMU_7	304593875.733	270066505.000	614061631
DMU_8	627942355.400	700372703.000	35473717
DMU_9	66382069.767	62336335.000	91266537
DMU_{10}	586948917.467	438426013.000	119274051
DMU_{11}	502591080.900	575795266.000	536047817
DMU_{12}	1327146530.367	1087740955.000	2130675856
DMU_{13}	498894530.933	460995173.500	38032494
DMU_{14}	481210293.633	449089904.500	508376348
DMU_{15}	360916244.467	369931846.000	527671
DMU_{16}	706029862.500	284024358.000	284024358
DMU_{17}	854730957.133	720651508.000	822247506
DMU_{18}	544403656.300	558834366.000	214798014
DMU_{19}	1005159006.600	1149866070.500	1393816922
DMU_{20}	2699708145.433	2480242411.500	4393062537

Results tabal (3-4)

	γ	τ
DMU_1	0.66	1.00
DMU_2	0.41	1.00
DMU_3	0.00	1.00
DMU_4	0.15	1.00
DMU_5	0.19	1.00
DMU_6	0.87	1.00
DMU_7	0.87	1.00
DMU_8	0.87	1.00
DMU_9	0.87	1.00
DMU_{10}	0.68	1.00
DMU_{11}	0.76	1.00
DMU_{12}	0.71	1.00
DMU_{13}	0.76	1.00
DMU_{14}	0.87	1.00
DMU_{15}	0.85	1.00
DMU_{16}	0.74	1.00
DMU_{17}	0.32	1.00
DMU_{18}	0.76	1.00
DMU_{19}	0.68	1.00
DMU_{20}	0.00	1.00