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A Novel Efficiency Ranking Approach Based on Goal Programming and Data Envelopment Analysis for the Evaluation of Iranian Banks

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Abstract
In the Iranian economy, banks play a key role in financing and developing the capital market. Therefore, it is important to evaluate the performance of stock banks. Data Envelopment Analysis (DEA) is a wide range of mathematical models used to measure the relative efficiency for a set of homogeneous decision-making units with similar inputs and outputs. In this paper, a novel efficiency ranking approach is proposed with two flexible mixed models derived from Goal Programming Data Envelopment Analysis (GPDEA) models. To solve this mismatch, we use Gantt chart to show DMUs' floating ranking and use another model to appoint the ranks exactly. In this paper, we analyze 18 stocks efficiency from bank industry of Tehran Exchange in 2016 using the GPDEA approach. Results demonstrate that the novel efficiency ranking approach has higher ability than the basic models in efficiency ranking.

Keywords: bank, Data Envelopment Analysis, efficiency, Goal Programming.

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1. Introduction

Financial institutions play a very important role in allocating resources, economic growth and job creation. For each country, the existence of financially viable companies is necessary to promote and support economic growth. Also, the banking is one of the most complex industries in the world and has a major share in the property and wealth of countries. In today's world, most banks are operating in a competitive and dynamic environment in which variables are constantly changing and it is difficult to predict these changes. On the other hand, banks spend a lot of time and money to achieve their goals.

To overcome this competitive environment, many bank officials and academic researchers have tried to find ways to improve the performance of banks. With increasing the foreign and domestic competition and the provision of diverse banking services and products, there is a serious need to improve the performance of the branches to stay competitive. Therefore, the durability of banks in a new competitive environment requires the existence of efficient branches' network. Due to the importance of this issue, the efficiency of active banks' branches in Iran has been measured and investigated. Data Envelopment Analysis (DEA) is a non-parametric method, can be solved by linear programming, and is nowadays widely used in most countries to evaluate the systems' performance with various activities such as maintaining airline bases, the police forces, banks, universities, insurance companies.

In 1990, Aly et al. present a non-parametric frontier approach to calculate the total, technical, pure technical, allocative, and scale efficiencies for a sample of 322 independent banks. The sample was drawn from the Federal Deposit Insurance Corporation tapes on the Reports of Condition and Reports of Income (Call Reports) for the year 1986.The main source of efficiency was technical in nature, rather than allocative and the results showed a low level of overall efficiency. Separate efficiency frontiers were constructed to test the effect of branching, however, the distributions of efficiency measures for branching and non-branching banks were not found to be different [1]. Miller and Noulas considered the relative technical efficiency of 201 large banks from 1984 to 1990 using DEA. In their study, averages of bank technical inefficiency were just over 5 percent, much lower than that was found in existing estimates and larger and more profitable banks have higher levels of technical efficiency [2]. Wheelock and Wilson reviewed the technical advances, inefficiencies and productivity changes in banking from 1984 to 1993. He used three inputs and five outputs. His main findings are that commercial banks experience reduced productivity, and they are technically more inefficient from 1984 to 1993 [3]. Bal and Orkcu have solved a multi-criteria data envelopment analysis (MCDEA) model, used in the literature to moderate the homogeneity of weights dispersion, using pre-emptive GP. The MCDEA model is solved using pre-emptive GP gives the same relative efficiency as the classical DEA model while it improves the homogeneity of input-output weights. This conclusion is confirmed by the computational results obtained when the two models are applied to a real data set relative to the socio-economic performances of European countries and their randomly generated instances with different numbers of decision making units, inputs and outputs [4].

Chiu and Chen examined the relationship between credit risk, market and operational performance with banks' performance. They used two methods of DEA and SFA (random border analysis method) and their results calculate the risk and efficiency of banks and finally, they found a significant relationship between risk and efficiency [5]. Holod and Lewis proposed an alternative DEA model for bank efficiency that treats deposits as an intermediate product, thus emphasizing the dual role of deposits in the bank production process. Consequently, the amount of deposits' effect on bank efficiency depends on the efficiency at both stages of the bank production process. The main advantage of their model was that it does not require a researcher to make a judgment call as to whether having more (production approach) or less (intermediation approach) deposits is ''better'' for bank efficiency. Their unified framework has the potential to produce more consistent efficiency estimates [6]. Halkos and Tzeremes have proposed a bootstrapped DEA-based procedure to pre-calculate and pre-evaluate the short-run operating efficiency gains of a potential bank merger or acquisition (M&A). They applied their proposed procedure to investigate the degree of operating efficiency gains of 45 possible bank M&As in the Greek banking over the period from 2007 to 2011 [7]. Bal et al. have developed two new models based on a multi-criteria data envelopment analysis (MCDEA) to moderate the homogeneity of weights distribution by using GP. These goal programming data envelopment analysis models, GPDEA-CCR and GPDEA-BCC, also improve the discrimination power of DEA [8].

Puri and Yadav have endeavored to propose a DEA model with undesirable outputs and further to extend it in fuzzy environment in view of the fact that input/output data are not always available in exact form in real life problems. They have proposed a fuzzy DEA model with undesirable fuzzy outputs which can be solved as a crisp linear program for (0, 1] using a-cut approach. Moreover, they have presented a numerical illustration followed by an application to the banking sector in India using fuzzy input/output data for the period 2009–2011 [9]. Tsolas and Giokas evaluated the efficiency of the branches of a large Greek bank using two GP and DEA methods. They use a minimal absolute deviation (a special case of GP / limited regression) and DEA as two performance measurement methods. Performance evaluation using GP is examined using two conceptual alternative models: one focusing on the transaction, and the other on the efficiency of production. The DEA evaluation has been done using the productivity model under a constant and variable return rate. The results confirm a very strong relationship between the GP and DEA rankings [10]. Moghadam et al. have sought to study and investigate about two methods for measuring efficiency: DEA and GP. The result of this study is an integrate DEA and the GP model is designed to find out the bank performance's weaknesses and alert the managers [11].

Johnes et al. compared Islamic banks with conventional banks in 18 countries between 2004 and 2009 using DEA. First, they compared the banks in terms of overall performance using the DEA method and concluded that there was no meaningful difference between the Islamic banks and the conventional banks in terms of efficiency, but when the efficiency decomposes into two concepts of functional efficiency and net efficiency, the results changed. In a performance that compares banks' performance on the basis of their banks' efficiency, it is concluded that there is a significant difference between Islamic banks and non-Islamic conventional banks, and Islamic banks in this sense of efficiency are less efficient. As a result, equilibrium between Islamic banks and conventional ones, and the efficiency of these banks are not significantly different [12]. Daneshvar et al. provide an integrated model of DEA and GP to improve the resolution, efficiency, and distribution of balanced and homogeneous weights by means of corrective models (GPDEA-CCR, GPDEA-BCC), and they provide the benefits of each of these corrective models with examples [13]. (Figure 1)

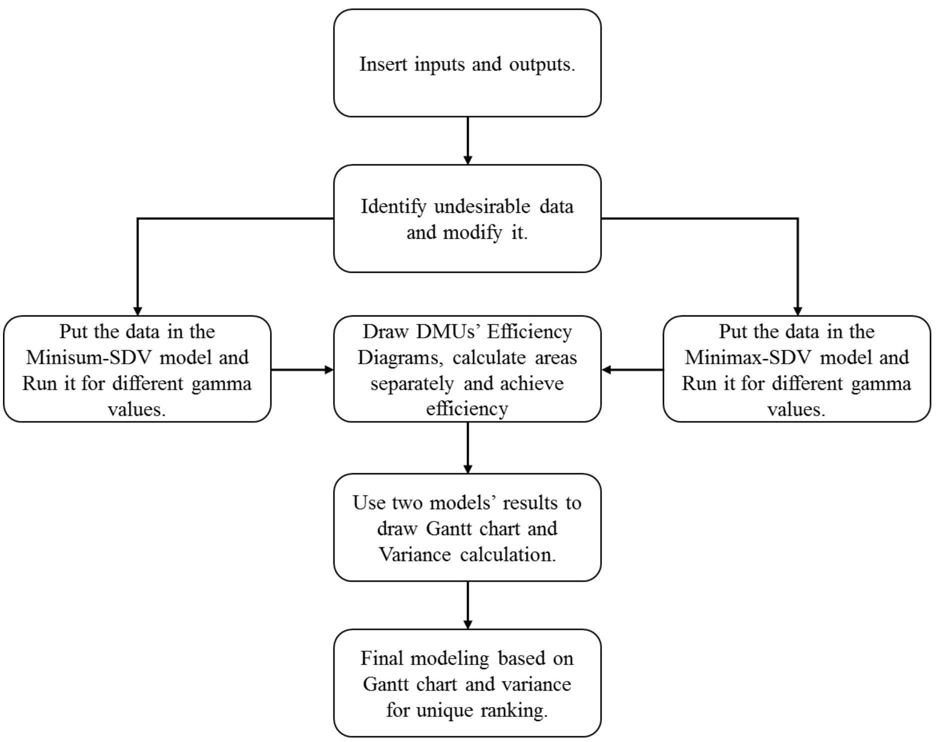
In this paper, we present a novel efficiency ranking approach. The proposed approach has two new flexible mixed models from three DEA basic models. The DEA basic models are based on Goal Programming. In order to Integration two models' results, first we draw DMUs' diagrams using multiple efficiencies, then calculate the graphs' area to achieve efficiency. But each proposed models give us unique efficiency which are not the same. To solve the mismatch problems, we use Gantt chart to show DMUs' floating ranking and use another model to appoint the ranks exactly.

In the next section, the theoretical framework is presented. The novel approach's mathematical modeling is introduced in Section 3. The information related to Iran's Banks is used for the proposed approach. In the fifth section, the results of this research have been studied.

2. Description of the theoretical framework

In this section, the theoretical framework for data envelopment analysis will be exhibited. First, the basic model of DEA is presented and then it is referred to due to the use of GP. Then, the basic DEA models based on GP are presented.

Figure 1. The diagram of research outline



2.1. Data Envelopment Analysis

Data envelopment analysis (DEA) is a technique for measuring the relative efficiencies of decision making units (DMUs), using similar inputs to produce similar outputs where the multiple inputs and outputs are incommensurate in nature. DEA has been one of the fastest growing areas of Operations Research and Management Science in the past decade. DEA has been applied to a wide variety of managerial and economic problem situations in both the public and private sectors [14].

The DEA model for evaluating the efficiency of a DMU established by Charnes et al. [15] is as follows:

$$\begin{aligned} & Max \quad h_0 = \sum_{r=1}^{s} u_r y_{r0} && \text{(I)} \\ & s.t. \\ & \sum_{i=1}^{m} v_i x_{i0} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 && j = 1,\dots,n, \\ & u_r, v_i \ge 0, && \text{for all } r \text{ and } i, \end{aligned}$$

where *j* is the DMU index, $j = 1,\dots,n$, *r* the output index, $r = 1,\dots,s$, *i* the input index, $i = 1,\dots,m$, y_{ri} the value of the output *r* for the DMU *j*, x_{ij} the value of the input *i* for the DMU *j*, u_r the weight given to the output *r*, v_i the weight given to the input *i*, and h_0 is the relative efficiency of unit under investigation, the DMU under evaluation. In this model, the unit under investigation is efficient if and only if $h_0 = 1$. There are many different DEA models. Most frequently to deal with the problems of discriminating power and weight restriction Model (I) is proposed. We will develop our proposed model based on model (I). There are some other multiple criteria approaches to DEA problems that are formulated based on different DEA models [16]. However, the focuses of those studies are somewhat different from ours.

Model (I) can be expressed equivalently in the following deviation variable form:

$$\begin{aligned} & Min \quad d_0 = 1 - \sum_{r=1}^{s} u_r y_{r0} && \text{(II)} \\ & s.t. \\ & \sum_{i=1}^{m} v_i x_{i0} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 && j = 1,\dots,n, \\ & u_r, v_i \ge 0, && \text{for all } r \text{ and } i, \end{aligned}$$

Where d_0 is the deviation variable for unit under investigation and d_j the deviation variable for the DMU *j* (appeared at the original inequality constraint *j*) in the model, the under investigation unit is efficient if and only if $d_0 = 0$ or, equivalently, $h_0 = 1$. If the unit under investigation is not efficient, its efficiency score is $h_0 = 1 - d_0$.

2.2. DEA and Goal Programming

In applying classical models, there are usually two disadvantages: weakness of resolution, and the unrealistic distribution of weights to the model's inputs and outputs. The weakness of resolution problem occurs when the number of units under evaluation is not large enough in comparison to the total number of inputs and outputs. The problem of irrational weights occurs when the weights assign a large-scale model to a single output or very small weights are assigned into a single input, which is illogical and undesirable. The data envelopment analysis model (is) based on goal programming versus the classical model has the higher ability to differentiate and present (s) the actual weights as follows.

2.2.1. Minimizing diversion variable model

Model (Π) could be presented as an ideal planning model as follows:

$$
\begin{aligned}
& Min\ \ d_0 && \text{(a)}\\
& \text{s.t:} && \\
& \sum_{i=1}^{m} v_i x_{i0} = 1 && \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 && j = 1,\dots,n\\
& u_r, v_i \geq 0 && \text{for all } r \text{ and } i,
\end{aligned}
$$

Where d_0 is the deviant variable for unit under investigation and d_j is the deviating variable for unit *j*. The value of d_0 in the range $[0,1)$ represents the Inefficiency value, the lower d_0, the inefficiency is less for a unit (and therefore more efficient), so it can be said that this classical model seeks to minimize inefficiency of the unit under investigation that is measured with d_0 [17].

2.2.2. Minimizing the total deviant variables model

Another method of Inefficiency measurement is a model that minimizes the total deviant variables. This model is called Minisum, and the general form of this model is as follows [18]:

$$
\begin{aligned}
& Min\ = \sum_{j=1}^{n} d_j && \text{(b)}\\
& \text{s.t:} && \\
& \sum_{i=1}^{m} v_i x_{i0} = 1 && \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 && j = 1,\dots,n\\
& u_r, v_i, d_j \geq 0 && \text{for all } r, i \text{ and } j
\end{aligned}
$$

2.2.3. Minimizing the maximum deviation model

If the maximum deviation value is indicated by M, its related mathematical relation can be written as $d_j \leq M \quad (j = 1,\dots,n)$. Now, if M is smaller and smaller, it means that the amount of deviations from the aspiration decreases. This model is called Minimax and is defined as follows [19]:

$$
\begin{aligned}
& Min\ = M && \text{(c)}\\
& \text{s.t.} && \\
& \sum_{i=1}^{m} v_i x_{i0} = 1 && \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 && j = 1,\dots,n\\
& M - d_j \geq 0 && j = 1,\dots,n\\
& u_r, v_i, d_j \geq 0 && \text{for all } r, i \text{ and } j
\end{aligned}
$$

2.3. Desirable or Undesirable Data

The general attitude in evaluating the efficiency of a unit is that reducing inputs and increasing outputs will improve performance. The CCR model is based on this attitude. However, it should be noted that organizations are not always seeking to maximize outputs and eliminate inputs because outputs and inputs can be desirable (good) or undesirable (bad). Another method is reducing unpredictable data in the model. If y_{rj}^{g} represents the desirable output (good) and y_{rj}^{b} represents the undesirable (bad) output, we want to increase y_{rj}^{g} and decrease y_{rj}^{b} to improve performance. Nevertheless, in multi-axis input-output models with constant output, both outputs y_{rj}^{b} and y_{rj}^{g} are increased to improve performance. Here, to increase the desirable output and reduce undesired output, we first multiply the undesired outputs in (-1), and then add the value of t_r to all negative outputs to make them positive, so that the following is true:

$$y_{rj}^{-b} = -y_{rj}^{g} + t_r > 0$$

And the value of t_r can be obtained from $t_r = Max\{y_{rj}^b\} + 1$. Other desirable outputs can be imported in the same way as before [20].

3. Novel Efficiency Ranking Approach

In models (a), (b) and (c), which based on a combination of Data Envelopment Analysis and Goal Programming, different results are obtained from each other. The sudden fluctuation and significant difference in the performance of a particular unit indicates the imbalance of these models. On the other hand, these models are unfairly determined by performance and ranking. They do not consider inter-model states. Modalities in which efficient units may be inefficient and inefficient units may be efficient. The novel efficiency ranking approach tries to investigate all existing states between the original GPDEA models that have not been addressed so far. The proposed approach has two new flexible mixed models from three basic GPDEA models. In order to Integration two models' results, first we draw DMUs' diagrams using multiple efficiencies, then calculate the graphs' area to achieve efficiency. But each proposed models give us unique efficiency which are not the same. To solve this mismatch, we use Gantt chart to show DMUs' floating ranking and we use another model to appoint the ranks exactly.

3.1. Proposed Flexible Mixed Models

Each of the models is found to be efficient in their unique benchmark. The model d_0 runs on the number of units, and the values obtained from it are not a realistic criterion for ranking the units, because each time it is executed, it tries to maximize the performance of the unit under consideration. In MinSum model, the sum of the variables of the deviation in the target function reaches its minimum value. In fact, it is simultaneously seeking to increase the efficiency of all units. In Minimax model, the maximum amount of deviation variability decreases. Typically, in this model, the performance is lower than two previous methods. In this paper, two more flexible models are presented, each of which is a combination of these basic models, with the difference that they provide more realistic performance.

Z_j is a zero-one variable ($j = 1, .., DMUs$). $Z_j = 1$ when d_j is summed up in the target function with other deviations, otherwise it will be zero. The *gamma* parameter is also defined to control the number of deviation variables in the model. The *gamma* will be at least equal to one and at most equal to the number of DMUs (i.e. $1 \le gama \le n$).

$$\text{Min} \sum_{j=1}^{n} Z_j d_j \tag{1}$$

s.t.

$$\sum_{j=1}^{n} Z_j = gamma \qquad gamma = 1, 2, ..., \text{n}$$

$$Z_0 = 1$$

$$\sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \qquad j = 1, ..., n$$

$$u_r v_i, d_j \ge 0 \text{ and } Z_j \in \{0,1\} \qquad \text{for all } r,\ i \text{ and } j$$

The objective function (1) minimizes the total deviation variables. Eq. (2) controls the number of deviation variables. So that each time the model is run, a certain number of variables are assigned to the target function. Eq. (3) ensures that each time the model is run; main unit under investigation is in the objective function. Eq. (4) and (5), like previous models, are the main constraints of DEA. Finally, decision variables are defined. We call the first proposed model by Minisum model with Some Deviation Variables (Minisum-SDV).

This model runs for every DIMU. Each

time the model is executed, a number of deviations are in the objective function. This number is equal to the gamma value. At each run, the deviation variable associated with the unit under investigation is surely in the target and among the other variables of deviation, the number of n-1 is selected. The method of selection is that the weights of inputs and outputs are selected in such a way that the least deviations are in the target function and their sum is minimized. This model fairly allows all DMUs to select the smallest deviation variables for different gamma values.

The second model we intend to offer will minimize the maximum deviation from a limited number of deviation variables. This model is a combination of two models, d_0 and Minimax, which considers the intermediate states of the two models.

$$\text{Min } z = M \tag{7}$$

s.t.

$$M \geq Z_j d_j \qquad j = 1, 2, \ldots, n \tag{8}$$

$$\sum_{j=1}^{n} Z_j = gamma \qquad gamma = 1, 2, \ldots, n \tag{9}$$

$$Z_0 = 1 \tag{10}$$

$$\sum_{i=1}^{m} v_i x_{i0} = 1 \tag{11}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \qquad j = 1, \ldots, n \tag{12}$$

$$u_r v_i, d_j \geq . \text{ and } Z_j \in \{0,1\} \qquad \text{for all } r, i \text{ and } j \tag{13}$$

The objective function (7) minimizes the maximum deviation variable or variables. Eq. (8) supports M to be more than all deviation variables. Eq. (9) controls the number of deviation variables. So that each time the model executes, a certain number of variables are considered into Eq. (8). Eq. (10) ensures that each time the model is run; main unit under investigation is in the objective function. Eq. (11) and Eq. (12), like previous models, are the main constraints of data envelopment analysis. Finally, decision variables are defined. We called the second proposed model by Minimax model with Some Deviation Variables (Minimax-SDV).

The model runs for every DMU. Each time the model is executed, the variable is at least equal to or greater than a number of deviations and attempts to minimize the maximum value of these variables. This number is equal to the gamma value. In each run, the variable is greater than or equal to the deviation variable for the unit under investigation. Among the other variables of deviation, the number of DMUs-1 is selected. The method of selection is that the weights of inputs and outputs are selected in such a way that the least deviations are included in the model.

These two proposed models are much more flexible than previous models, and calculates the efficiency for all diversion variables. This flexibility is determined by the *gamma* parameter. The Minisum-SDV model turns into the model (a), when *gamma=1* and turns into model (b), when *gamma* = number of DMUs. But the Minimax-SDV model turns into the model (a), when *gamma=1* and turns into model (c), when *gamma* = number of DMUs. This flexibility and coverage of the base models.

3.2. Achieve unique DMUs efficiency

It is enough to use this method to achieve unique efficiency for DMUs. First we draw DMUs' diagrams using multiple efficiencies, then we calculate the graphs' area to achieve efficiency. But each proposed models give us unique efficiency which are not the same. To solve the mismatch problems, we use Gantt chart to show DMUs' floating ranking and use another model to appoint the ranks exactly.

3.2.1. Drawing DMUs' Efficiency Diagrams

The results of the model show DMUs performance per bank for different gamma. While our goal was to introduce unique efficiencies to provide a proper ranking of banks. To solve this problem, taking into account for each bank DMUs' efficiencies has been obtained, we can draw the performance chart of each bank. In these diagrams, the horizontal axis shows gamma variations and vertical axis shows efficiency. After drawing the DMUs' efficiencies charts, we should calculate their areas.

They can be parsed in simple geometric shapes, and the area of each graph can be obtained from the sum of its smaller components. However, it is better to use computational software such as MATLAB to eliminate the error generated by manual calculations.

The maximum area occurs when the efficiency of one unit for all *gamma* is one, i.e. for all gamers, to be effective. In case of being efficient, for all gamma values, the area will be 1 * (DMUs-1). But we know the numerical efficiency is [0, 1], so we use the following equation:

$efficiency_i = \frac{bank_i's\ area}{maximum\ area}.$

After achieving the efficiencies, we can rank the DMUs. But each model give us unique ranks that is different from another. To solve the problem, we will integrate two models' ranking.

3.2.2. Gantt chart and Variance calculation

After achieving the efficiency, it is need to use integration ranks. In order to achieve this goal, we have used Gantt chart. Gantt chart is a type of bar chart that illustrates a project schedule. Gantt charts illustrate the start and finish dates of the terminal elements and summary elements of a project. Terminal elements and summary elements comprise the work breakdown structure of the project. Modern Gantt charts also show the dependency (i.e., precedence network) relationships between activities. Gantt charts can be used to show current schedule status using percent-complete shadings and a vertical "TODAY" line as shown here. Although now they are regarded as a common charting technique, Gantt charts were considered revolutionary when first introduced. This chart is also used in information technology to represent the collected data.

The solutions obtained from two models that were presented before can be the same; in this case, there is no interval and a unique efficiency is obtained. But in many cases, they are different. The numbers that are ranked as a rating for a given unit can be represented by an interval. For example, a unit can get rank 10 via the Minisum-SDV model, and rank 14 via the Minimax-SDV model. Therefore, this unit has a rating of [10, 14]. These intervals can be displayed in a Gantt chart. The intervals for each unit are determined by this, then we obtain each rank's variance. In other words, for each unit, we calculate the ratings variance that may be assigned to it. It tries to select the less rank's variance. For example, in the interval [10, 14], rank 12 has the lowest variance between 10, 11, 13, and 14. But it is not possible to choose the midrange with the least variance for all the intervals, and this is a big challenge.

In order to calculate variance, If interval would be $J_j = [a_j, b_j]$, we have presented $\sigma_{jt}{}^2 = \frac{\sum_{i \in J_j} (rank_t - rank_i)^2}{n_j}$ for $j = 1,2, \ldots, DMUs$ and $rank_t = a_j, a_j + 1, \ldots, b_j$.

3.2.3. Final modeling and unique ranking

In this section, using a linear model, a unique ranking is obtained. The ranking obtained from this model has the least variance in the ranking; of course, it is the rank of variance, not the efficiency variance. τ_{jt} is binary variables, when

$\tau_{jt} = 1$, unit j has rank t, and in otherwise is zero. The linear model for the minimization of variance is as follows:

$$Min\ Z = \sum_j \sum_t \sigma_{jt}^2 * \tau_{jt} \qquad (14)$$

s.t.

$$\sum_j \tau_{jt} = 1 \qquad for\ all\ t \in J_j \qquad (15)$$

$$\sum_t \tau_{jt} = 1 \qquad for\ all\ j = 1,\ldots,n. \qquad (16)$$

$$\tau_{jt} \epsilon \{0,1\} \qquad for\ all\ j = 1,\ldots,n \ \ and\ \ t \in J_j \qquad (17)$$

The (14) statement, as an objective function, tries to reduce the total variances. Eq. (15) ensures that the rank of each interval is selected as the unique rank. Eq. (16) supports all units in the ranking, in other words, assigns one to each units. Finally, the binary variable is defined. In the next part, with the presentation of a case study, the proposed approach will be used and evaluated.

4. Case Study

Due to the role of banking system in economic growth, unemployment and inflation control, the banking system is one of the most important economic pillars in many countries. Therefore, efficiency analysis is considered as a suitable measure for assessing the firms' efficiency in this industry. One of the most important issues that make senior executives of banks often fail to implement programs and efficiency evaluation methods in their organization is the program's disproportion to their needs. On the other hand, with increasing the disclosure of the monetary development crucial role and monetary markets, especially banks in support of the real sector and ultimately, the development and prosperity of the economy, the banks financial system's reform has become more concern for the country makers attention. Therefore, it was imperative to evaluate this industry by employing basic DEA models with GP and the proposed models. For this reason, 18 banks that have been most welcomed by the Iranian banks are being evaluated.

But one of the problems in determining inputs and outputs is that in most cases there is no information about them, which makes it difficult to decide on inputs and outputs in the studies that are about the efficiency evaluation of banking units by DEA method. Two important factors in the selection of input and output variables are effective: research purpose and, statistical constraints and sample size. Regarding the literature review, with the subject of evaluating the efficiency of bank branches by DEA method and viewing inputs and outputs of these papers, we considered four inputs and two outputs. The input and output variables are introduced in Table 1. Now, the data of each Iranian bank for 2016 is based on four outputs: number of personnel, number of branches, capital (billion rials) and, general and administrative costs (billion rials), and two inputs: net profit (billion rials) and risk. In this case, the risk is an undesirable output because we seek to reduce it, not increase it. To solve this problem, we consider reversal of risk values in place of risk in the output. But in net profit, there are some negative numbers that represent losses. To overcome this problem, another method is used to reduce unpredictable data in the model. After solving the problem of undesired data, the data is presented in Table 2. After solving the problem of undesired data, we put the data in A, B, and C models. The efficiency of each bank in each model is shown in Table 3 and Figure 2.

As shown in Figure 2, some banks, such as "Gardeshgari", "Parsian", "Hekmat Iranian", "Khavarmianeh", "Day", "Sina" and "Karafarin", have the least fluctuation in performance and indeed the differences between models. But other banks have a lot of fluctuations and declaring a specific number as efficiency is very difficult. On the other hand, the numbers of performance affect the ranking and assessment of banks. For example, Iran Zamin Bank based on model (a) has performance 1 and rank 1, it has a performance of 0.44 and a rating of 5 based on model (b) and it has a performance of 0.023 and a rating of 14 based on model (c),. The key question is that "what is the efficiency and rating for this bank?"

As stated, the underlying problem is that we do not know how much for banks efficiency should be introduced, since in most cases, the performance based on each model is different with other models. On the other hand, these models are unfairly determined by efficiency and ranking. They do not consider inter-model states. Modalities in which efficient units may be inefficient and inefficient units may be efficient. The novel efficiency ranking approach tries to investigate all existing states between the original GPDEA models that have not been addressed so far. The purpose of these two proposed models is to study the floating mode between the basic models. Also, based on a new structure, we present unique efficiency from each proposed models.

Table 4 and Table 5 show the results of solving the Minisum-SDV model for different gamma values for all units and Table 6 and Table 7 show the results of solving the Minimax-SDV model, too. Minisum-SDV models are run by GAMS for 324 times.The results of the model show 18 efficiencies per banks for different *gamma*. While our goal was to introduce a number as an efficiency to provide a proper ranking of banks. To solve this problem, taking into account that for each bank 18 efficiency has been obtained, we can draw the performance chart of each bank. In these diagrams, the horizontal axis, gamma variations and vertical axis shows efficiency. The bank efficiency charts for *gamma* variations are plotted in Figure 3 and Figure 4.

To achieve an efficiency for each bank, the area under each bank's graph is calculated. The maximum area occurs when the efficiency of a unit for all gamma is one. In case of being efficient, for all gamma values, the area will be 1 * (18-1). But we know that the numerical efficiency is [0,1], so we use the following equation: $efficiency_i = \frac{bank_i's\ area}{maximum\ area}$. The area of each chart and the efficiency of each bank are shown in Table 8.

Given the rankings obtained from the Minisum-SDV and Minimax-SDV models, the rank of each unit is obtained. After that, we draw on Gantt chart. The horizontal axis of the Gantt chart shows the rankings, and the horizontal axis shows the units under study. In addition, the variance of each rank is computed and included in the Gantt chart in Figure 5.

In the Gantt chart, for example: the Gavamin Bank, if ranked 10th, will have a size of 1.7 variances, and if ranked 11th, it will have a variance of 0.7 and if ranked 12th, has 1.7 variances. Similarly, analysis can be done for other banks. Furthermore commentation on this item is considered worthwhile. The intervals were extracted from the two models of part 3.1 and displayed in the Gantt chart. We are now going to the linear model presented in part 3.2.2. As previously stated, this model tries to give a unique rating to each unit.

The model is easily coded in the Lingo software. The results of the model provide a unique ranking for each unit. The value of the objective function is $Z^* = 33.2$, which is the optimum level of variance. Based on the variables that have taken the value of one, it is determined banks ranking. The ranking of the banks is shown in Table 9. In models (a), (b) and (c), which base models are a combination of Data Envelopment Analysis and Goal Programming, different performance of the units under study. The sudden fluctuation and significant difference in

the performance of a particular unit indicates the imbalance of these models.
On the other hand, these models are unfairly determined by performance and ranking. They do not consider inter-model states. Modalities in which efficient units may be inefficient and inefficient units may be efficient. The novel efficiency ranking approach tries to investigate all existing states between the original GPDEA models that have not been addressed so far.
The proposed approach seeks to genuinely determine the ratings of decision-making units in order to obtain an accurate picture of the status quo. In the new approach to performance rating, floating modes are checked between base models to avoid bias in ranking. In this approach, for the change of the gama parameter, which depends on the number of DMUs, all possible modes for determining the efficiency are examined. And for each unit, performance is achieved for all units. After charting each unit, calculating the area of the sub-graph and normalizing it, the unit performance is obtained for each of the Minisum-SDV and Minimax-SDV models. But there is still a multi-level problem for each unit. In order to solve this problem, the rankings obtained from the two suggested models create intervals. These intervals are plotted in Gantt chart, and the variance of each rank is calculated for each unit. After extraction of variances, this time is placed in another linear model. This model tries to select ratings with the least variance from the average of the defined interval. In the end, the ratings of the banks are realistic and taking into account all possible modes.

5. Conclusion

Since the introduction of the Data Envelopment Analysis Method by Charles et al. , this method has become an effective tool for evaluation and modeling. In this method, the relative efficiency of each decision making unit is equal to the rational ratio of outputs to inputs. The disadvantage of this method is that the number of units evaluated is related to the number of input and output variables. That is, the higher the number of problem variables, the base models have less power to distinguish between efficient and non-working units. Also, when the number of organizational units is less than a certain amount, the power of differentiation of the basic models of data envelopment analysis decreases. In this research, based on the concepts of the ideal planning technique, a model is proposed to improve the assessment of the decision-making units effeciency . The model can solve some of the problems of data envelopment analysis models, including the weakness of the resolution of decision-making units, and in this regard, increases the efficiency of these models. To test this model and compare it with the basic model, information from 18 banks with four input vriables and two output variables were used.
In the proposed approach for changing the gama parameter, which depends on the number of DMUs, all possible modes for determining the efficiency are examined. And for each unit, performance is achieved for all units. After charting each unit, calculating the area of the sub-graph and normalizing it, the unit performance is obtained for each of the Minisum-SDV and Minimax-SDV models. But there is still a multi-level problem for each unit. In order to solve this problem, the rankings obtained from the two suggested models create intervals. These intervals are plotted in Gantt chart, and the variance of each rank is calculated for each unit. After extraction of variances, this time is placed in another linear model. This model tries to select ratings with the least variance from the average of the defined interval. In the end, the ratings of the banks were realistic, taking into account all possible modes. The “Khavarmianeh” Bank had the highest efficiency and was ranked first. The

"Mellat" bank also achieved the lowest rank, the eighteenth rank, due to the lowest efficiency level.

For future research, it is suggested that private banks and government banks be compared. Other models of data envelopment analysis, such as network and output models, should be used for current studies and the results should be presented using the proposed model. We hope this research can be used to evaluate different units.

Moreover, uncertain DEA models such as fuzzy DEA, and robust DEA can be applied for performance measurement of banks in the presence of uncertainty [21]-[26].

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Table 1. Input & Output Variables

Inputs	number of Staff	x_1
	number of branches	x_2
	Capital	x_3
	General and administrative costs	x_4
outputs	net profit	y_1
	Risk	y_2

Table 2. Banks' data

Row	Banks	Inputs				Outputs	
		Nu. of Staff	Nu. of branches	capital	costs	net profit	risk
1	Ghavamin	6,783	734	5,182	8,000	683	0.478
2	Gardeshgari	1,055	83	1,433	6,000	143	0.240
3	Eghtesad novin	3,169	282	4,668	11,312	21,269	1.075
4	Ansar	5,217	636	3,775	8,000	2,650	0.885
5	Iran zamin	2,183	327	941	4,000	26,877	0.346
6	Parsian	4,405	294	5,556	23,760	1,820	0.690
7	Pasargad	3,815	327	6,495	50,400	12,047	0.971
8	Tejarat	18,734	1,661	10,171	45,700	24,499	0.280
9	Hekmat iranian	1,137	132	1,053	4,000	828	0.752
10	Khavarmianeh	349	16	615	4,000	2,023	0.917
11	Day	1,007	91	2,819	6,400	2,031	0.549
12	Saman	2,475	143	3,708	8,000	824	0.281
13	Sarmayeh	1,349	143	1,215	4,000	26,303	0.476
14	Sina	2,405	257	2,765	10,000	1,638	0.714
15	Saderat	30,676	2,578	32,742	57,800	1	0.787
16	Mellat	21,177	1,581	37,763	50,000	3,982	0.258
17	Karafarin	1,908	107	1,945	8,500	604	1.124
18	Post bank of Iran	2,952	406	2,946	3,233	2	0.312

Table 3. Banks' efficiency and ranking under basic models

Banks	Model(a)	Ranking(a)	Model(b)	Ranking(b)	Model(c)	Ranking(c)
Ghavamin	0.319	8	0.015	13	0.014	17
Gardeshgari	0.213	10	0.212	7	0.213	7
Eghtesad novin	0.607	3	0.607	3	0.506	4
Ansar	0.6	4	0.043	12	0.03	13
Iran zamin	1	1	0.44	5	0.023	14
Parsian	0.157	12	0.157	8	0.155	9
Pasargad	0.244	9	0.115	9	0.103	10
Tejarat	0.105	13	0.063	11	0.033	12
Hekmat iranian	1	1	1	1	1	1
Khavarmianeh	1	1	1	1	1	1
Day	0.469	6	0.469	4	0.456	5
Saman	0.191	11	0.065	10	0.187	8
Sarmayeh	1	1	1	1	0.633	3
Sina	0.385	7	0.385	6	0.38	6
Saderat	0.072	14	0.011	15	0.072	11
Mellat	0.032	15	0.008	16	0.022	15
Karafarin	0.703	2	0.699	2	0.703	2
Post bank of Iran	0.513	5	0.012	14	0.016	16

Figure 2. Banks' Efficiency under basic models

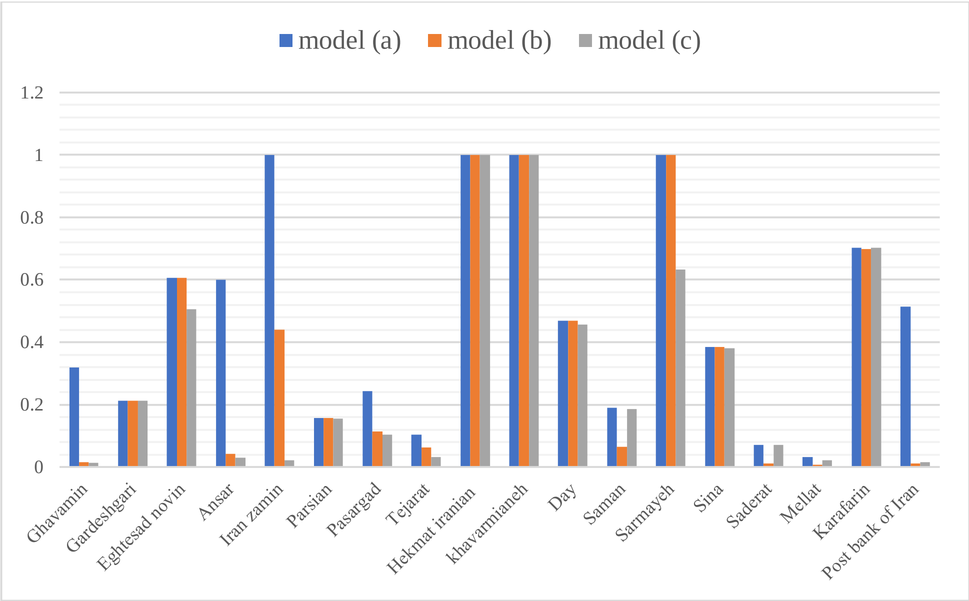


Table 4. Banks' efficiency under Minisum-SDV

Gamma	Ghavamin	Gardeshgari	Eghtesad novin	Ansar	Iran zamin	Parsian	Pasargad	Tejarat	Hekmat irania
2	0.319	0.212	0.607	0.6	1	0.157	0.244	0.105	1
3	0.319	0.212	0.607	0.6	1	0.157	0.244	0.096	1
4	0.319	0.212	0.607	0.6	0.837	0.157	0.115	0.063	1
5	0.319	0.212	0.607	0.6	0.837	0.157	0.134	0.078	1
6	0.319	0.212	0.607	0.6	0.837	0.157	0.115	0.063	1
7	0.319	0.212	0.607	0.6	0.837	0.157	0.115	0.063	1
8	0.319	0.212	0.607	0.6	0.837	0.157	0.115	0.063	1
9	0.319	0.212	0.607	0.6	0.837	0.157	0.115	0.063	1
10	0.015	0.212	0.607	0.6	0.837	0.157	0.115	0.063	1
11	0.015	0.212	0.607	0.6	0.837	0.157	0.115	0.063	1
12	0.015	0.212	0.607	0.6	0.837	0.157	0.115	0.063	1
13	0.015	0.212	0.607	0.043	0.44	0.157	0.115	0.063	1
14	0.015	0.212	0.607	0.043	0.44	0.157	0.115	0.063	1
15	0.015	0.212	0.295	0.043	0.44	0.157	0.115	0.063	1
16	0.015	0.212	0.295	0.043	0.44	0.157	0.115	0.063	1
17	0.015	0.212	0.607	0.043	0.44	0.157	0.115	0.063	1
18	0.015	0.212	0.607	0.043	0.44	0.157	0.115	0.063	1

Table 5. Banks' efficiency under Minisum-SDV

Gamma	khavarmianeh	Day	Saman	Sarmayeh	Sina	Saderat	Mellat	Karafarin	Post bank of Iran
2	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.507
3	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.507
4	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.507
5	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.507
6	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.211
7	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.507
8	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.507
9	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.012
10	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.012
11	1	0.469	0.191	1	0.385	0.072	0.008	0.699	0.012
12	1	0.469	0.191	1	0.385	0.072	0.008	0.699	0.012
13	1	0.469	0.191	1	0.385	0.072	0.008	0.699	0.012
14	1	0.469	0.191	1	0.385	0.072	0.008	0.699	0.012
15	1	0.172	0.065	1	0.385	0.011	0.008	0.699	0.012
16	1	0.172	0.065	1	0.385	0.017	0.008	0.699	0.012
17	1	0.172	0.065	1	0.385	0.019	0.008	0.699	0.012
18	1	0.469	0.065	1	0.385	0.011	0.008	0.699	0.012

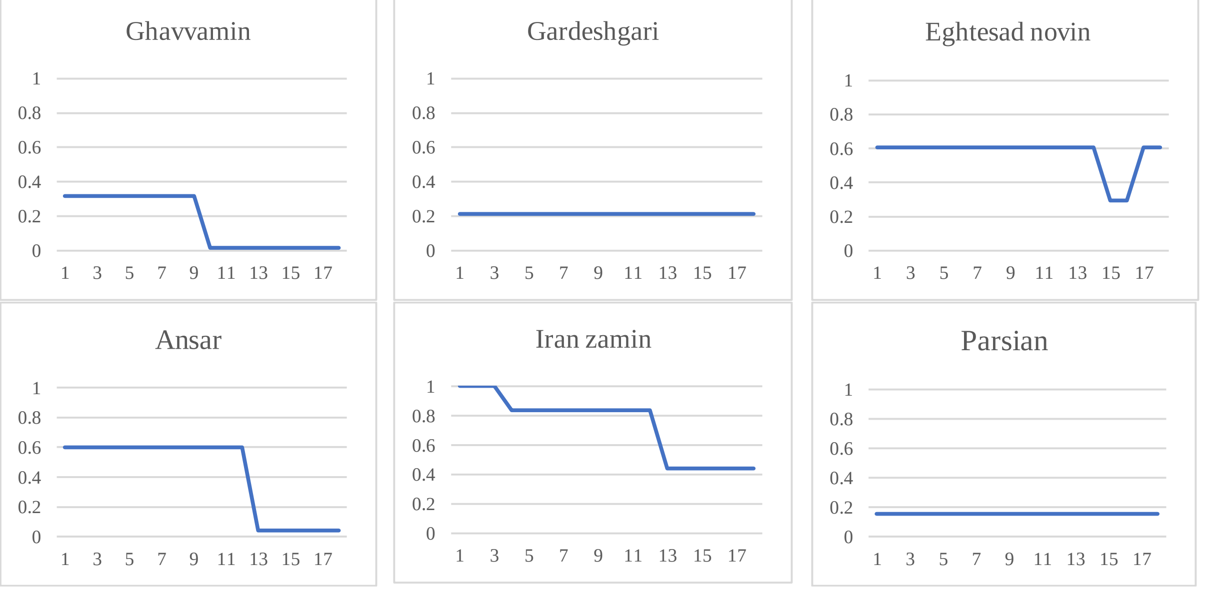
Table 6. Banks' efficiency under Minimax-SDV

Gamma	Ghavamin	Gardeshgari	Eghtesad novin	Ansar	Iran zamin	Parsian	Pasargad	Tejarat	Hekmat iranian
2	0.319	0.213	0.607	0.6	0.587	0.157	0.244	0.105	1
3	0.319	0.213	0.607	0.5	1	0.157	0.244	0.105	1
4	0.319	0.212	0.607	0.6	0.909	0.157	0.195	0.105	0.911
5	0.319	0.212	0.607	0.6	0.605	0.157	0.181	0.105	1
6	0.319	0.163	0.31	0.199	0.579	0.157	0.181	0.105	0.958
7	0.319	0.131	0.335	0.117	0.562	0.105	0.181	0.105	1
8	0.319	0.004	0.16	0.03	0.472	0.012	0.118	0.105	0.989
9	0.319	0.004	0.16	0.03	0.472	0.012	0.118	0.105	0.333
10	0.319	0.004	0.16	0.03	0.44	0.012	0.205	0.105	0.923
11	0.005	0.004	0.16	0.03	0.44	0.012	0.181	0.105	0.121
12	0.005	0.004	0.16	0.386	0.44	0.012	0.181	0.105	0.121
13	0.005	0.212	0.6	0.533	0.457	0.012	0.181	0.105	0.234
14	0.091	0.163	0.31	0.124	0.478	0.012	0.118	0.105	0.546
15	0.069	0.131	0.335	0.117	0.491	0.012	0.181	0.105	0.357
16	0.005	0.004	0.16	0.03	0.447	0.012	0.118	0.105	0.028
17	0.015	0.212	0.607	0.043	0.44	0.157	0.115	0.079	0.121
18	0.014	0.213	0.506	0.03	0.023	0.155	0.103	0.033	1

Table 7. Banks' efficiency under Minimax-SDV

Gamma	khavarmianeh	Day	Saman	Sarmayeh	Sina	Saderat	Mellat	Karafarin	Post bank of Iran
2	1	0.366	0.191	0.454	0.385	0.072	0.032	0.477	0.239
3	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.042
4	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.056
5	1	0.469	0.191	0.774	0.385	0.072	0.032	0.699	0.507
6	1	0.469	0.191	0.654	0.385	0.072	0.032	0.699	0.507
7	1	0.172	0.065	1	0.244	0.072	0.032	0.45	0.11
8	1	0.469	0.066	1	0.207	0.072	0.032	0.432	0.239
9	1	0.025	0.008	1	0.021	0.072	0.032	0.011	0.213
10	1	0.025	0.008	1	0.021	0.072	0.032	0.011	0.163
11	1	0.025	0.008	1	0.021	0.072	0.032	0.699	0.122
12	1	0.438	0.008	1	0.021	0.072	0.032	0.699	0.252
13	1	0.469	0.191	1	0.021	0.072	0.032	0.699	0.056
14	1	0.172	0.065	1	0.244	0.072	0.032	0.327	0.054
15	1	0.177	0.066	1	0.207	0.072	0.032	0.432	0.056
16	1	0.025	0.008	0.758	0.021	0.072	0.032	0.011	0
17	1	0.469	0.191	1	0.385	0.072	0.032	0.699	0.012
18	1	0.456	0.187	0.633	0.38	0.072	0.022	0.703	0.016

Figure 3. Bank efficiency charts under Minisum-SDV model



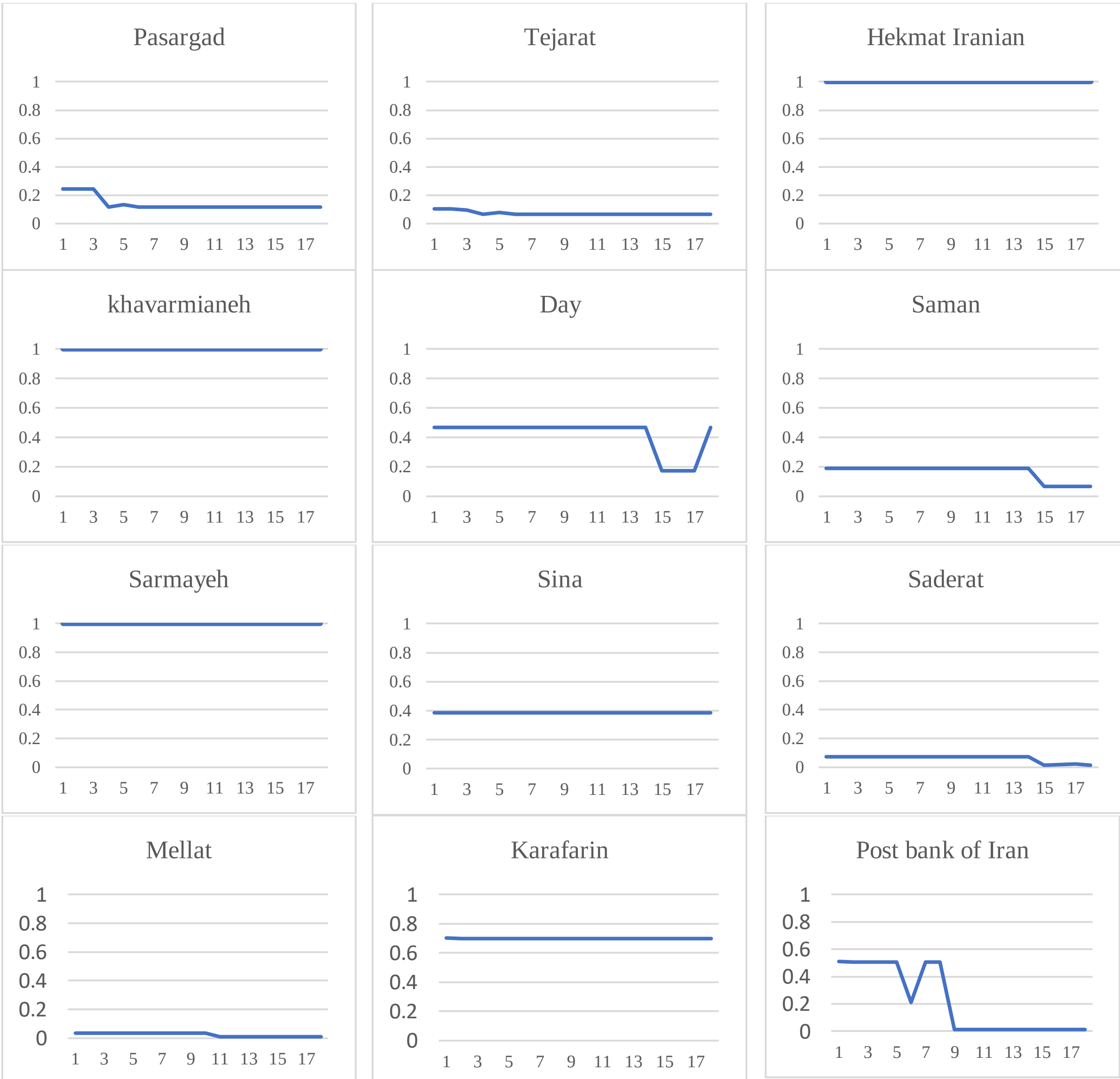
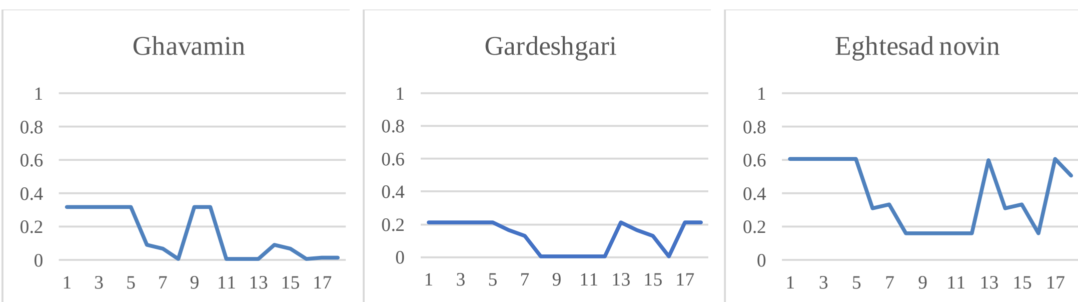


Figure 4. Bank efficiency charts under Minimax-SDV model



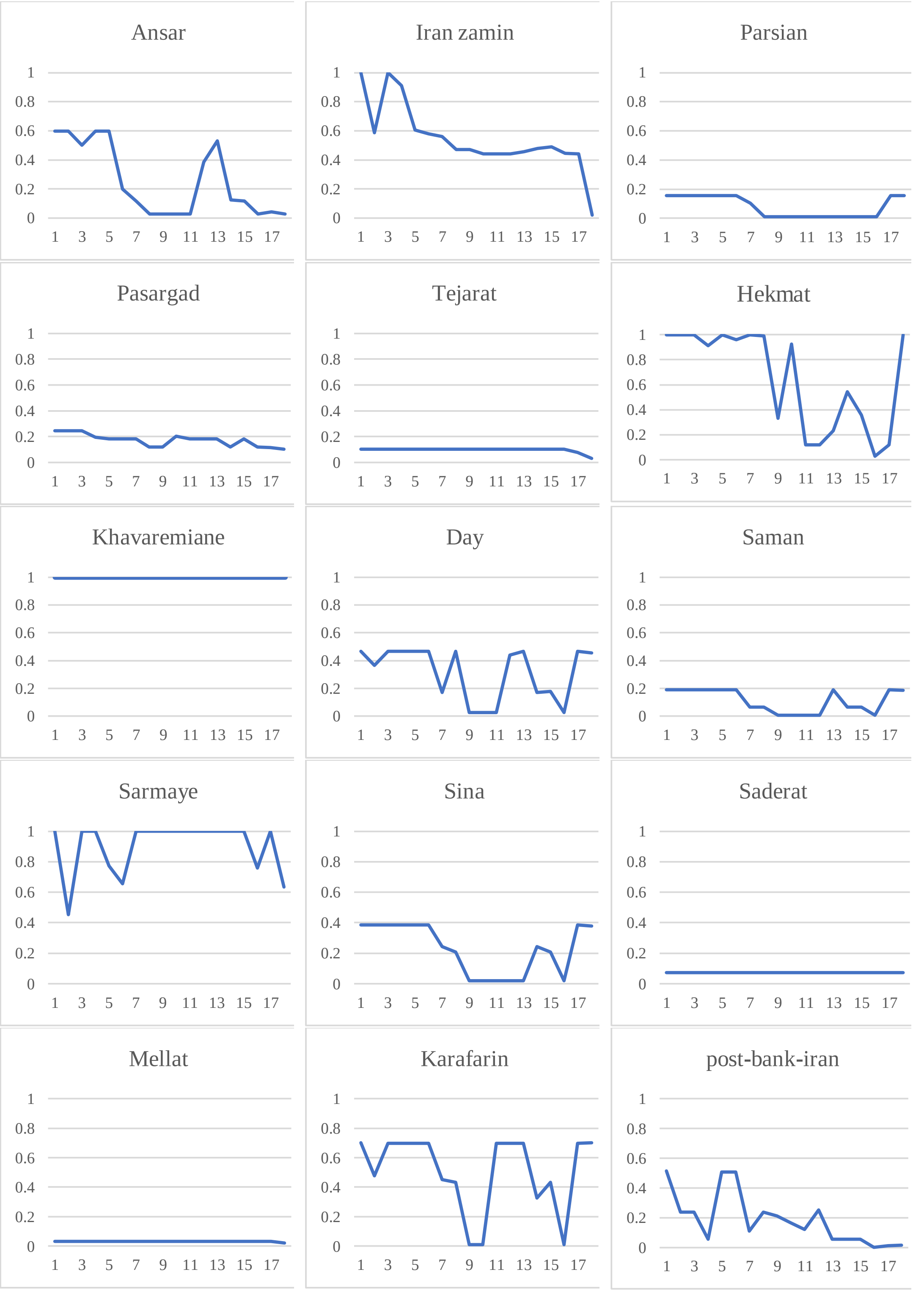

Ansar
Iran zamin
Parsian
Pasargad
Tejarat
Hekmat
Khavaremiane
Day
Saman
Sarmaye
Sina
Saderat
Mellat
Karafarin
post-bank-iran
0
0.2
0.4
0.6
0.8
1
1 3 5 7 9 11 13 15 17

Table 8. The area of each chart and the efficiency of each bank under Minisum-SDV and Minimax-SDV

Banks	Minisum-SDV			Minimax-SDV		
	area	**efficiency**	**rank**	**area**	**efficiency**	**rank**
Ghavamin	2.839	0.167	10	2.38	0.14	12
Gardeshgari	3.604	0.212	8	2.091	0.123	13
Eghtesad Novin	8.993	0.529	4	6.443	0.379	6
Ansar	7.089	0.417	5	4.284	0.252	8
Iran Zamin	12.444	0.732	2	9.333	0.549	4
Parsian	2.669	0.157	12	1.309	0.077	16
Pasargad	2.244	0.132	13	2.567	0.151	11
Tejarat	1.547	0.091	14	1.717	0.101	15
Hekmat iranian	17	1	1	10.727	0.631	3
khavarmianeh	17	1	1	17	1	1
Day	6.732	0.396	6	5.168	0.304	7
Saman	2.805	0.165	11	1.819	0.107	14
Sarmayeh	17	1	1	15.691	0.923	2
Sina	6.545	0.385	7	3.536	0.208	9
Saderat	1.037	0.061	15	1.224	0.072	17
Mellat	0.357	0.021	16	0.527	0.031	18
Karafarin	11.883	0.699	3	8.415	0.495	5
Post bank of Iran	3.604	0.212	9	2.839	0.167	10

Figure 5. Gantt chart

Banks	Interval ranking	Ranks																	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ghavamin	[10,12]										1.67	0.67	1.67						
Gardeshgari	[8,13]								9.17	5.17	3.17	3.17	5.17	9.17					
Eghtesad novin	[4,6]				1.67	0.67	1.67												
Ansar	[5,8]					3.5	1.5	1.5	3.5										
Iran zamin	[2,4]		1.67	0.67	1.67														
Parsian	[12,16]												6	3	2	3	6		
Pasargad	[13,11]											1.67	0.67	1.67					
Tejarat	[14,15]														0.5	0.5			
Hekmat iranian	[1,3]	1.67	0.67	1.67															
khavarmianeh	[1,1]	0																	
Day	[6,7]						0.5	0.5											
Saman	[11,14]											3.5	1.5	1.5	3.5				
Sarmayeh	[1,2]	0.5	0.5																
Sina	[7,9]							1.67	0.67	1.67									
Saderat	[15,17]															1.67	0.67	1.67	
Mellat	[16,18]																1.67	0.67	1.67
Karafarin	[3,5]			1.67	0.67	1.67													
Post bank of Iran	[9,10]									0.5	0.5								

Table 9. Final banks' ranking

Banks	Final Ranking
khavarmianeh	1
Sarmayeh	2
Hekmat iranian	3
Iran zamin	4
Karafarin	5
Eghtesad novin	6
Day	7
Ansar	8
Sina	9
Post bank of Iran	10
Gardeshgari	11
Ghavamin	12
Pasargad	13
Saman	14
Tejarat	15
Parsian	16
Saderat	17
Mellat	18