Comparison of the Accuracy of Black Hole Algorithms and Gravitational Research and the Hybrid Method in Portfolio Optimization

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ABSTRACT

The main purpose of this research is portfolio optimization in Tehran securities exchange using the black hole algorithm and the Gravitational Research algorithm. We also propose an algorithm named Hybrid Algorithm which combines the two algorithms above to cover the weaknesses of these two algorithms. Finally we compare the results with the Markowitz model and choose the optimal algorithm.

In order to analyze the data that is the same information extracted from the TSE Client software and RahAvard Novin Software, MATLAB software version of 2016 and GAMS and SPSS have been used.

This research is fulfilled in the period from 2011 to 2016. The method used in this study is based on the purpose of the applied research and based on the way of data collection as a descriptive research and correlation type, which is noticed with the retrospective and post-event approach and through the analysis of the observed information, attempts to optimize the portfolio using a black hole algorithm. In all the years of research, the hybrid method introduced in this research has obtained the nearest solution to the exact solution, which is the same Markowitz. In order to optimize the portfolio, black hole meta-heuristic algorithms, Gravitational Research and hybrid algorithm (hybrid) can be used instead of the Markowitz algorithm with higher accuracy and speed. The results of the present case study and other studies show that black hole algorithms, Gravitational Research, and hybrid algorithms are very quick in solving portfolio optimization problems

Keywords:
Black hole algorithm, Gravitational Research algorithm, Hybrid algorithm, Portfolio Optimization.

1. Introduction

Decision-making is one of the most important tasks of managers and most of the time they spend decision making on different issues. It is believed that the essence of management is to make a proper decision and usually management have the same meaning with decision-making. In an apropos definition, decision-making is: Choosing a solution among different practical solutions to solve problems and using opportunities (Sinaei & Zamani, 2015). The decision-making process is one of the most complex processes of human thought that several factors and methods have played a role, and each may have a different result. Orasano, J., & Connolly, T. (1993) defined decision making as a series of cognitive activities which is made by science (Orasano & Connolly, 1993)

Narayan (2001) introduced the decision-making as follows: decision-making is an interaction between the problem that needs to be solved and a person hoping to solve the problem with the specific context of the issue (Narayan, 2001) .Managers in different organization, especially in organizations whose main activities are in financial affairs, always faced with different options to make decisions, and their decision making at the right time with adequate accuracy is accounted as a key factor for the success of their organizations. Investment decisions, particularly the portfolios formation, are known as essential and critical decisions to organizations (Sinaei & Zamani, 2015).

Nowadays, the method of choosing of stocks in the securities exchange for investors is supposed to be one of the main concerns in these markets, and the choice of stock or portfolio which is the best in terms of profitability, price increments and earnings per share is of great importance. The issue of choosing an optimal portfolio is one that all investors, including real and legal, encounter. This issue involves creating of a portfolio that maximizes the investor’s utility. For this purpose, many methods have been developed and introduced in relation to the portfolio selection. The majority of these methods have used information and financial analysis for the proper selection and decision-making (Abadian & Shajari, 2017).

In fact, the concept of increasing the power of stock market prediction and diversification is considered as the main factor in development and financial decision-making. This important concept of Harry Markowitz's theory as portfolio selection was determined in financial literature. The attention on portfolio risk and portfolio returns is one of the principles of Markowitz at the same time for the investor. The idea that financial decision- making, arises from the interaction between portfolio risk and portfolio returns, created a revolution in investment management for two reasons: First, it supposes that an investor conducts a quantitative assessment of portfolio risk and portfolio returns by considering the portfolio returns and simultaneous movement of portfolios of returns with regard to each other, which is the main idea of portfolio diversification. Second, it focuses on the financial decision-making process as an optimization issue, i.e. the investor selects the portfolio among with the least variance among the various types of available portfolios (Rahnama Roodpashti et.al, 2016).

To decide on the selection of a portfolio related to single-period portfolios, the assumption is that the investor decided to allocate assets for once and for N existing assets at the beginning of the expected period (e.g. one season or one year), based on The risk and the relationships between returns, within that horizon.

Decision-making is made only once and is not allowed to review until the end of the period, and the effect of decisions on followed periods is not taken into consideration. Moreover this model was based on three restrictive assumptions: (1) the short-term investing horizons. (2) The cost of transaction in the market has not been considered. 3) The problem parameters are deterministic and already known. While portfolios are a few more general and multi-cycle portfolios, the investor makes a decision and each decision effects on subsequent decisions. The purpose of finding the decision for settlement in each period and taking into account a set of future change opportunities (the availability of assets and risk features and their returns), are the remaining investment horizon, final transaction costs, and other constraints (Najafi & Moushekhian, 2015).

Since Markowitz has published his model, this model created many changes and improvement in the way people approach to capitalization and portfolios and has been used as an efficient tool to optimize portfolios. Decision-making related to portfolio is complicated because several variables should be considered in this case. Variables such as rate of return on capital, earnings per share, price ratio to earnings per share, risk appetite, and other factors. Since a set of variables is considered for decision-making, one must use multi-criteria decision making. The data envelopment analysis method, which is one of the multi-criteria decision-making methods, makes it possible to be done. Based on this method, the best options can be determined (Azar, Khosravani, & Jalali, 2014).The emergence of the areas of liberty and the removal of some of the disadvantaged regulations in the economies of countries over past decades, in the emergence of first signs, abandonment of the fixed exchange rate system in the early 1970th, caused financial markets to face more fluctuations in price variables. Also the emergence of monetary unions and the globalization of the economy have led to a Sharpe increase in transitivity of financial crises from one market to another, which means an increase in fluctuations in financial markets. On the other hand, the Sharpe decline in stock prices in many capital markets has resulted in significant losses for the active elements in them. Therefore, activity in financial markets will be accompanied by a lack of confidence and risk, and is important for investors to measure the risk of in different portfolios. Meanwhile, measuring and evaluating undesirable risks for financial institutions and capital market actors is of particular importance. (Asgharpour et.al, 2015).

The present study, using a combination of these methods, a new combination in the field of financial literature, tries to find a new and meta-heuristic answer to the problem-solving of the efficient Horizons search with the average Semi-variance approach. In addition to examining the classical and heuristic methods of optimization, this research combines heuristic algorithms and applies it to the portfolios optimization in the Tehran securities Exchange among the top 50 companies in market.

2. Literature Review

The first steps of theory building in portfolio optimization were taken by Markowitz. Before presenting the average of variance of Markowitz, risk was considered a qualitative factor and was not used in the calculations. In 1963 William Sharpe presented a single-index model. By explaining the sensitivity factor as risk, Sharpe eliminated the limitations of CAPM and CMT application for each single paper and the limitation of application about empirical cases. The above three models can be described as the most important step in modeling the portfolio optimization problem. Other research also added more realistic constraints to the primary models. Studies have also been devoted to solving the problem with new and more efficient methods that can be grouped into two categories of precise algorithms and heuristic and meta-heuristic algorithms. For the first time in a comprehensive study in 2000, Chang et al. showed that meta-heuristic algorithms can be used to solve discontinuous problem. (Chang et.al, 2000)

In an article entitled "selecting a portfolio from Tehran securities Exchange among accepted companies using the Genetic Algorithm Optimization Model", Modares , Mohammadi estakhri (2009) found out that there is a significant difference and a remarkable superiority in the results of the genetic algorithm method for the portfolios 10, 20 and 30 with respect to the first portfolio of the random method for the risk and return variables. (Modares & Mohammadi Estakhri, 2009)

Raei et al. (2012) in an article entitled of "The portfolio Optimization using the average Semi-variance approach and using the Harmonic Search method", stated that variance is usually considered as the general risk appetite factor to create an Efficient frontier and form optimal portfolio. But since Semi-variance presents a better estimate of the real risk appetite of a portfolio, Semi-variance is considered as the main risk appetite factor in this research. The matter of portfolio optimization is about a combination of the integer number programming and class 2 programming and there is no specific and efficient algorithm to solve such problems. (Raei & Ali Beigi, 2012)

Shalchi (2012), by examining portfolio optimization with consolidated approach of data envelopment analysis and exploratory factor analysis, claimed that the issue of portfolio optimization is one of the most important and attractive matter in the financial and investment issues. In this research, a mathematical model is proposed to form optimal portfolio. So in this model, stock weights possess a unique feature. The mentioned weights include efficient indices which show different aspects of financial performance of companies and different financial indices, represent independent indices for comparing companies and also provide independent indices for comparing companies and ranking them. It is worth noting that first the effective financial indices are extracted from literature review in the selection of portfolios, and their significance is determined by the investment experts' perspective. Then, an explanatory factor analysis is used to reduce the dimension of problem and eliminate the correlation between these indices. Moreover, Topsis model have been used to obtain the stock weight. (Shalchi, 2012)

Najafi and Moushkhian (2015) presented a model in a research entitled "Modeling and providing an optimal solution for multi-period investment portfolio optimization with genetic algorithm" in order to overcome the expressed constraints and bring it closer to the real world. Hence in the following, a model known as the multi-period investment portfolio optimization of value of average Semi-variance, provided a transaction with considering the costs and after modeling, this model was solved using the genetic algorithm. In this study, in order to solve the model the data of 24 stocks of Tehran Securities Exchange (TSE) were used as model inputs from January 2008 to December 2013. The results have shown that this algorithm is suitable for solving such problems and has the necessary efficiency. (Najafi & Moushekhian, 2015)

In a study entitled "Increasing the power of prediction of stock market using flexible planning", Khanjarpanah et al. (2016) provide a new model based on modern portfolio theory and add constraints such as the number of stocks and the flexibility of stock weight in the portfolio. In the application of the stock weight flexibility limitation, there is a state in which constraint satisfactory, flexibility and uncertainty are used to model the flexibility of fuzzy relations. In addition to stated uncertainty, returns on stock also have cognitive uncertainty. So fuzzy approach has been used to deal with uncertainty, both of them are flexible and possible programming (fuzzy programming subsets). Also the fuzzy approach has been used to transform the model into a simple problem. (Khanjarpanah et al, 2016)

Rombouts and Verbeek (2009) used daily returns on stock related to financial indices of 500 S&P and Nasdaq the value of risk in the framework of the multi-variable GARCH model and formed optimized portfolio. The prediction of the value exposed to risk of stocks and consideration of the calculated optimal weight and the comparison of them with real losses have shown that the degree of failure was high. (Rombouts & Verbeek, 2009)

In 2011, Woodside Oriakhi used three genetic algorithms, prohibited searches and simulated annealing to determine the effective frontier of the Markowitz model. The point in all the research is that no model researchers have proposed a model beyond the Chung model, and only achieved a comparison and development of the methods to find an effective and proper frontier with better performance. (Woodside Oriakhi, 2011)

Chen et al. (2011) proposed a Prometheus Language method to decide on investment portfolios. At first quantitative and qualitative data were collected for each stock, and in the continuous this method used to determine the investment portfolio. Finally, an example is given to explain the eligibility of the proposed method. (Chen, Hung, & Cheng, 2011)

Vetschera & De Almeida(2012) studied and examined the use of Promethean ranking method to resolve issues related to portfolio selection. In this research, a new formula for portfolio selection was developed based on Prometheus method. (Vetschera & De Almeida, 2012)

Dewandaru et al. (2014) construct an active Islamic portfolio using a rotational and multi-style strategy derived from three important styles, namely, movement, value, and investment quality. In this study, the stock which was consistently listed in the Islamic indices of Dow Jones existing in the United States in 1996-2012, was utilized. Also, two large economic portfolios supplemented to receive monthly insurance premiums from industrial production growth and innovation of inflation, compliance of economic regime changes. Based in the information coefficients, a movement with duration of six months and fractal measurement as acceleration factors, company performance (gross profit), and market ratio as assessment factors, return of capital and all ranked accrual items as quality factors were adopted. The existing of an active portfolio using EBLF model to prevent problems of was carried out by displaying the predicted factors by means of middleware maker Markov Switching of Bayesian. The presentation of proposed portfolio produces 0.7-0.8 data on the composite indices and the 0.42-0.48 data ratio on the stylistic indices, with an annual alpha of 11%-10. Even when a limited error of 1% was placed on the benchmark, the portfolio still produces 0.9-1.2 data ratio before transactions cost, and 0.6-0.8 after transactions cost (Dewandaru et al, 2014)

Farag (2015) explores the impact of price constraints on extreme reaction in emerging markets (Evidence from the Egyptian stock market). The purpose of this study was to investigate the effect of price constraints on the extreme reaction hypothesis in the Egyptian Securities Exchange during the period of 2010-1999. The price return observed in the two and three days and above and below the price constraint will cause the further move of stock price at the initial price, and this research confirmed the hypothesis of the directional effects of price movement of large companies by the price of the reverse stock price. (Farag, 2015)

Weng Siew et al. (2015) study the effect of human behavior towards different level of risk appetite for portfolio selection and this study examines the different levels of risk appetite of people by using a developed decision-making model. The results of the survey were conducted in Malaysia during the years 2010 to 2013 and the 23 weekly stock prices survey of the Malaysian Securities exchange. The results showed that human behavior is effective in determining the different levels of risk appetite in selecting different portfolios . (Weng Siew et al, 2015)

Peng Zhang (2018) proposed a new multiperiod mean absolute deviation uncertain chance-constrained portfolio selection model with transaction costs, borrowing constraints, threshold constraints and cardinality constraints. The Results of the survey were conduct in Shanghai Stock Exchange. The result showed that Based on uncertain theories, the model is converted to a dynamic optimization problem. Because of the transaction costs and cardinality constraints, the multiperiod portfolio selection is a mix integer dynamic optimization problem with path dependence, which is "NP hard" problem. The proposed model is approximated to a mix integer dynamic programming model. A novel discrete iteration method is designed to obtain the optimal portfolio strategy, and is proved linearly convergent. (Zhang, 2018)

Wei Chen, Yun Wang, Pankaj Gupta, Mukesh Kumar Mehlawat (2018) discussed a portfolio selection problem under the mean-variance-skewness framework wherein the security returns are obtained through evaluation of the experts instead of historical data. y treating security returns as the uncertain variables, an uncertain mean-variance-skewness model is proposed for portfolio selection under consideration of the transaction costs, bounds on holdings, cardinality of the portfolio, and minimum transaction lots constraints. To solve the resultant portfolio selection problem, which is an NP-Complete nonlinear integer programming problem, a hybrid solution method termed the FA-GA is developed by combining features of the firefly algorithm (FA) and genetic algorithm (GA). In the proposed method, the crossover and mutation operators of the GA are integrated into the FA to strike an optimal balance between the exploration and exploitation. (Chen et al, 2018)

Salehpoor ,Molla-Alizadeh-Zavardeh (2019) developed a method based on mean–variance (MV) , mean absolute deviation (MAD), semi variance (SV) and variance with skewness (VWS). The developed algorithms are Electromagnetism-like algorithm (EM), particle swarm optimization (PSO), genetic algorithm (GA), genetic network programming (GNP) and simulated annealing (SA). Also a diversification mechanism strategy is implemented and hybridized with the developed algorithms to increase the diversity and overcome local optimality. The sustainability of this proposed model is verified by 50 factories on the Iranian stock exchange. Finally, experimental results of proposed algorithms with cardinality constraint are compared with each other by four effective metrics in which the algorithms performance for achieving the optimal solution discussed. In addition, they have done the analysis of variance technique to confirm the validity and accurately analyze of the results which the success of this method was proved. (Salehpour & Molla Alizadeh Zavardehi, 2019)

3. Methodology

The method utilized in this research is in terms of practical purpose and survey hypothesis testing of correlation type and data collection using historical information as "post-event" means the use of past information. Stock price information has been obtained through the TSE Clint and RahAvard Novin software. Information about the theoretical issues has also been collected from library resources, including books, journals and specialized finance and accounting management sites.

3.1. Research Hypotheses

With regard to wideness and diversity of meta-heuristic algorithm which have been used more than before, one question arises to know whether black hole algorithms and Gravitational Research have the ability to optimize the portfolios.

Due to the optimization problem in this thesis, we do not have hypothesis in this study and we are seeking the ability to optimize the portfolio by black hole algorithms and Gravitational Research and then compare the ability of these two algorithms. Also we introduce another algorithm from the combination of optimal advantages of each of these two algorithms, which we call hybrid algorithm. In the following, we will examine this algorithm in terms of the ability to optimize portfolios.

If we want to consider the hypotheses for this research, these can be considered as follows:

Hypothesis 1: a meta-heuristic black hole algorithm is capable of optimizing the portfolio in Tehran's leading stock companies.

Hypothesis 2: Gravitational Research meta-heuristic algorithm is able to optimize portfolio in Tehran's leading stock companies.

Hypothesis 3: a meta-heuristic hybrid black hole algorithm-Gravitational Research is capable of optimizing the portfolio in Tehran's leading stock companies.

First, a number of companies that were listed in the top 50 stock companies in the years 1999 to 2004 was prepared. This list will be submitted every three months by the Security Exchange. Then the companies that were on the list in all four periods of one year were selected. After this level, the stock price data of these companies was obtained by the TSE Client and the RahAvard Novin software.

Portfolio performance is measured using efficiency and risk. The average return and risk is formulated as:

(1)

$$r_p = \sum r_i w_i$$

(2)

$$TE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}(R_{Pt} - R_{It})^2}$$

R_{Pt} is the average return on the portfolio at time t. R_{It}Indicates the return on the indices at time t. $r_p$is return on portfolio and r_i is return on stock i, L_i and U_i Upper and lower frontier of stock fluctuations of i and shows the number of shares in the portfolio. V_{iT} shows The price of each unit of stock i at time t. x_i indicates the number of units of stock i in the portfolio. w_i displays the the weight of the stock in the portfolio. c shows the total amount of capital. TE represents the risk measurement against the benchmark index. α Returns, more than the benchmark, makes the index proiminent (Islami Bidgoli & Kordlouie, 2010)

3.2. Models

The Black Hole Algorithm (BH) is an innovative method inspired by the natural black hole phenomenon. Due to its simplicity and ease of implementation, it has attracted much attention and has been used since its inception to solve many practical optimization problems. The black hole algorithm is an evolutionary algorithm that was introduced by Hatamloo in 2013 and was first used on the data clustering problem. This algorithm has been inspired by the black hole phenomenon and, similar to other population-based algorithms, the BH algorithm begins with the initial population of candidate solutions for an issue and optimizes the objective function for which is calculated for them. In each replication, the black hole algorithm is considered the best candidate and then starts to pull out other candidates around it, called the star. If a star gets closer to a black hole, it will be swallowed by the black hole and will disappear forever. In this case, a new star (candidate solution) is generated randomly and placed in search space and starts searching for a new one. Like other population-based algorithms, in the black hole the population of candidate solutions (stars) is randomly generated and placed in the search space of a problems or function. After initialization, the fitness value of population, the evaluation, and the best candidate in the community which has the best fit, is chosen as black hole, and the rest give a natural star formation. The black hole has the ability to capture the stars that surround it. After initialization of the black hole and stars, the black hole begins to attract the star around it, and all the stars begin to move toward the black hole. This attraction of the star by the black hole is formulated as follows:

$$x_i(t+1) = x_i(t) + rand * \left(x_{BH} - x_i(t)\right) \quad i = 1,2,\ldots,N \quad (3$$

In the formula for the star i in the repetition of t, t + 1 is in order. The location of the black hole is in the search space. Rand is a random number in interval [0,1]. N is the number of stars (the candidate solution).

In a star's motion toward the black hole, a star may reach a location at a cost below the black hole. So the black hole moves to star's location and vice versa. Then the BH algorithm will continue with the black hole in the new location, and then the stars will start moving toward this new location. Moreover, there is the possibility of passing the event horizon in the motion of the stars towards the black hole. Each star that passes through the black hole event horizon is sucked off by a black hole. Every time a candidate (star) dies it is sucked by a black hole. Another candidate solution is born and randomly distributed in the search space and starts a new search. After moving all the stars, the next is repeated. The radius of the event horizon in the black hole algorithm is calculated using the follow equation:

(4)
$$R = \frac{f_{BH}}{\sum_{i=1}^{N} f_i}$$

In the formula, ، $f_{BH}$is the value of the fitness of the black hole. And $f_i$is the fitness of the star i. N is the number of stars (the candidate solution), When distance, a candidate solution and the black hole (the best candidate), is less than R. that candidate falls and a new candidate is created and is randomly distributed in the search space.

Gravitational Research Algorithm (GSA) is an optimization method based on collective intelligence which is presented with the inspiration of concepts of mass and gravity and simulation of related rules. This algorithm searches for a multi-dimensional search space to find the minimum value of the objective function. In GSA, search factors are considered as mass existed in the space and their efficiency is measured by their mass. All of these masses absorb each other by gravitational force, and this causes them to move toward the heavier mass. The heavier masses fit the good answers are slower than the rest. $rand_i$ is a uniform random variable in the range [0,1], which is used to maintain the random property of search. It is used to set the gravity constant from the exponential relation $G(t) = G_0 e^{-\alpha \frac{t}{T}}$which is in fact the convergent factor of the algorithm over the elapsed repetition of the algorithm. Here, α and G_0 are considered constant and t, T respectively represent the total repetitions and the current repetition.

Mass values are calculated by assessing merit. A heavier mass is a more effective factor. This means that better masses are more attractive and move slowly.

(5)
$$m_i(t) = \frac{fit_i(t) - worst\ (t)}{best\ (t) - worst\ (t)} \qquad M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)}$$

In the above equation, the fitness value$fit_i(t)$ is the evaluation function of i particle at time t. Also, worst (t), (best (t), which indicates the fitness of the strongest and weakest particles of the population at time t, obtained using the following relationships for a minimization problem.

In optimization algorithms, exploration refers to the ability to search for unknown different areas of the space in the solution space in order to fine global optimum, while extraction refers to the ability to utilize the knowledge of the prior desirable solutions to find better solutions. Therefore, in order to obtain desired performance, the exact adjustment is necessary between extraction and exploration properties.

The black hole algorithm can be extracted and the Gravitational Research algorithm is capable of optimum exploration. The main idea of the hybrid algorithm is to combine the ability of the black hole (best candidate) in BH with the method of searching based on candidate's mass (better fitness) at the GSA for moving candidates. The details of the proposed hybrid algorithm are as follows: First, all factors are randomly initialized. Each factor is considered as a solution. After initialization, the gravitational constant, the gravitational force and the resulted force calculated between the factors. After that, particle acceleration is obtained. In each repetition, the best candidate (black hole) is determined. After calculating the acceleration and updating the next candidate speed, the next space of the particle is calculated based on the following relation:

(6)
$$x_i(t+1) = x_i(t) + rand * \left(x_{BH} - x_i(t)\right) + v_i\ (t+1)$$
$$i = 1,2, \ldots\ldots, N$$

In order to analyze the data that is extracted from the TSE Client and RahAvard Novin software, Matlab software version of 2016 and GAMS and SPSS have been used.

Step One: First, the list of top 50 companies was obtained for the three-month periods in the years 90-95 (2012-2017). Subsequently, for each year, companies selected in each of the four rounds were selected. Using the information extracted from the TSE Client software, the new stock prices were obtained in daily intervals for a period of 4 years.

The second step: We entered the data in the 2016 version of MATLAB software, and we obtained the return and variance, and the Semi-variance and covariance and the rest of the requirements in the algorithms.

The third step: We solved algorithms for each year independently and drew their charts.

The fourth step: We entered the data obtained in the second step in GAMS software, and we optimized for each year with the Markowitz method.

The fifth step: We compared the data from the third and fourth steps, and we derived the average distance of each algorithm from the Markowitz algorithm.

The sixth step: Using the software and SPSS, we test the data and confirm or reject the hypotheses.

Below is a descriptive statistic table for 2011-2012. Below is a descriptive statistics table for 2011-2012.

Table 1 - Descriptive data table of 2011-2016

1390																
	beterans	pardis	dejaber	segharb	sefars	fazar	vabemelat	vabimeh	vatoosa	vasakht	vasepah	vasina	vasanat	vaghadir		
Average	-0.07%	-0.14%	-0.09%	0.25%	-0.05%	-0.03%	-0.14%	0.02%	-0.06%	0.16%	-0.12%	0.01%	-0.01%	-0.14%		
Variance	2.13%	0.08%	0.04%	0.14%	0.04%	0.07%	0.05%	0.07%	0.04%	0.07%	0.07%	0.10%	0.04%	0.26%		
Standard Dev	14.61%	2.75%	2.06%	3.77%	2.03%	2.71%	2.34%	2.73%	2.07%	2.57%	2.69%	3.20%	2.06%	5.15%		
Kurt	11364.58%	3141.83%	96.84%	5088.49%	1051.81%	-54.36%	6744.14%	-68.44%	22.14%	-57.48%	3668.48%	5182.11%	-11.48%	16478.06%		
Max	149.30%	4.80%	5.85%	31.51%	5.84%	5.48%	5.30%	6.80%	5.64%	6.77%	6.14%	28.76%	5.13%	6.02%		
Min	-165.96%	-26.01%	-7.66%	-35.31%	-14.52%	-8.80%	-26.66%	-5.58%	-5.72%	-6.96%	-26.38%	-28.45%	-4.84%	-72.73%		
Skew	-163.72%	-350.77%	29.61%	-103.85%	-130.24%	-1.02%	-577.79%	16.17%	23.40%	4.02%	-379.74%	15.34%	10.13%	-1164.04%		
Semi Variance	14.65%	2.77%	1.22%	3.21%	1.69%	1.52%	2.53%	1.54%	1.22%	1.44%	2.65%	2.64%	1.24%	7.71%		
VAR	-3.74%	-3.74%	-3.20%	-3.05%	-2.98%	-4.06%	-2.51%	-4.20%	-3.31%	-3.80%	-3.70%	-3.12%	-3.28%	-3.29%		
1391																
	beterans	senoosa	retko	sefars	ghazar	fabahonar	foolad	kechini	kehram	vabeshahr	vapars	vasina	vasanat	vaghadir	valesapa	
Average	-0.12%	0.03%	-0.08%	0.06%	0.12%	0.09%	0.20%	-0.05%	-0.16%	-0.12%	0.01%	-0.09%	0.04%	-0.05%	-0.44%	
Variance	0.06%	0.08%	0.08%	0.06%	0.50%	0.09%	0.07%	0.12%	0.08%	0.06%	0.03%	0.29%	0.06%	0.07%	0.25%	
Standard Dev	2.46%	2.81%	2.84%	2.41%	7.07%	2.93%	2.61%	3.44%	2.89%	2.53%	1.60%	5.35%	2.42%	2.59%	4.98%	
Kurt	-58.97%	143.92%	-95.17%	91.26%	6723.06%	495.39%	751.58%	3250.21%	-106.21%	1750.56%	101.78%	9286.64%	902.12%	1761.31%	6212.48%	
Max	5.65%	12.28%	6.13%	9.85%	72.91%	17.15%	13.39%	25.66%	5.63%	7.75%	5.47%	55.05%	14.19%	6.17%	52.71%	
Min	-7.27%	-8.32%	-7.34%	-6.09%	-59.22%	-12.84%	-14.04%	-29.04%	-6.63%	-17.48%	-4.47%	-54.98%	-14.45%	-20.61%	-32.29%	
Skew	9.91%	34.08%	9.07%	36.28%	225.62%	62.91%	45.87%	-64.79%	-1.62%	-235.11%	23.38%	6.29%	2.80%	-232.28%	381.53%	
Semi Variance	1.29%	1.86%	1.47%	1.36%	5.27%	1.66%	1.66%	2.73%	1.52%	2.49%	1.02%	5.28%	1.67%	2.28%	3.59%	
VAR	-3.90%	-4.90%	-4.07%	-3.67%	-4.65%	-3.84%	-3.28%	-3.45%	-4.26%	-3.22%	-2.38%	-3.13%	-3.42%	-3.01%	-4.01%	
1392																
	akhaber	khodro	remapna	sefars	shebandar	shebehran	shefan	shiran	fakhas	fameli	vabeshahr	vabemelat	vapasar	vatejarat	vakharazm	
Average	0.15%	0.47%	0.56%	0.47%	0.39%	0.45%	0.32%	-0.23%	0.05%	0.02%	0.33%	0.30%	0.32%	0.84%	0.16%	
Variance	0.07%	0.14%	0.06%	0.09%	0.19%	0.06%	0.11%	0.30%	0.03%	0.32%	0.09%	0.06%	0.06%	0.45%	0.16%	
Standard Dev	2.59%	3.71%	2.51%	2.94%	4.41%	2.41%	3.33%	5.50%	1.64%	5.62%	2.93%	2.43%	2.48%	6.72%	4.01%	
Kurt	811.82%	1469.00%	444.62%	-80.20%	14016.19%	1278.06%	2316.06%	15201.03%	769.67%	8113.33%	277.94%	1924.74%	901.19%	16712.52%	8612.54%	
Max	6.43%	26.48%	7.49%	7.63%	60.17%	17.95%	22.15%	18.41%	7.91%	50.82%	16.78%	5.74%	11.59%	96.04%	5.93%	
Min	-17.47%	-7.12%	-14.59%	-9.05%	-6.70%	-4.74%	-25.55%	-76.36%	-7.31%	-60.18%	-7.00%	-20.04%	-16.44%	-11.26%	-48.07%	
Skew	-122.64%	251.98%	-82.05%	-9.50%	1030.43%	219.39%	-53.88%	-1076.87%	-11.82%	-185.23%	62.81%	-228.06%	-77.79%	1174.36%	-723.39%	
Semi Variance	2.00%	1.74%	1.78%	1.60%	1.34%	1.19%	2.55%	7.43%	1.54%	5.24%	1.55%	2.06%	1.84%	1.71%	4.39%	
VAR	-3.63%	-4.22%	-3.51%	-4.06%	-3.19%	-2.96%	-3.78%	-3.28%	-2.11%	-3.20%	-3.76%	-3.11%	-3.57%	-3.79%	-3.70%	
1393																
	parsan	sharak	shaspa	fars	fakhooz	fameli	foolad	kachad	vaomid	vaansar	vapasar	vakharazm	vasapa	vasandogh		
Average	-0.49%	-0.67%	1.03%	-0.18%	-0.23%	-0.10%	-0.16%	-0.36%	-0.09%	0.04%	-0.21%	-0.14%	-0.06%	-0.07%		
Variance	0.27%	0.69%	3.43%	0.03%	0.03%	0.05%	0.08%	0.04%	0.02%	0.06%	0.07%	0.04%	0.08%	0.09%		
Standard Dev	5.20%	8.32%	18.51%	1.63%	1.64%	2.33%	2.86%	2.07%	1.35%	2.41%	2.64%	2.03%	2.86%	3.00%		
Kurt	19084.28%	21571.85%	23658.83%	253.19%	1759.75%	1718.22%	3594.56%	2279.97%	196.72%	290.69%	8977.92%	-34.28%	1676.16%	4852.05%		
Max	5.17%	7.43%	286.22%	7.32%	5.75%	12.84%	23.05%	6.79%	4.89%	11.13%	4.70%	4.47%	7.93%	27.66%		
Min	-76.42%	-126.00%	-4.41%	-4.62%	-11.88%	-17.85%	-19.74%	-17.39%	-4.05%	-11.70%	-32.27%	-4.92%	-23.23%	-24.95%		
Skew	-1295.12%	-1420.61%	1523.15%	56.27%	-174.26%	-102.39%	138.51%	-255.22%	50.29%	8.39%	-734.77%	19.86%	-201.46%	100.39%		
Semi Variance	7.60%	12.45%	1.08%	1.01%	1.60%	1.96%	2.20%	1.96%	0.86%	1.48%	3.15%	1.14%	2.38%	2.40%		
VAR	-2.76%	-2.79%	-2.46%	-2.85%	-2.09%	-2.71%	-2.36%	-2.58%	-2.26%	-3.33%	-2.81%	-3.39%	-4.00%	-3.39%		
1394																
	hekeshti	khebahman	khesapa	khodro	shekhark	fazar	fakhooz	foolad	kachad	vatejarat	vasandogh	valesapa	vamaaden			
Average	0.22%	0.25%	0.42%	0.24%	-0.06%	0.22%	0.05%	0.04%	0.03%	0.10%	0.29%	0.17%	0.02%			
Variance	0.05%	0.09%	0.15%	0.08%	0.07%	0.07%	0.24%	0.16%	0.04%	0.06%	0.12%	0.06%	0.16%			
Standard Dev	2.22%	2.96%	3.93%	2.89%	2.71%	2.58%	4.94%	3.98%	1.94%	2.37%	3.53%	2.46%	4.00%			
Kurt	-26.37%	1553.22%	9369.32%	290.99%	5207.84%	-37.64%	3025.24%	8004.64%	582.50%	911.37%	5265.62%	-5.80%	3405.57%			
Max	6.87%	23.56%	48.73%	10.79%	22.35%	6.79%	34.90%	27.96%	9.98%	12.87%	35.05%	7.80%	28.19%			
Min	-4.96%	-11.62%	-4.96%	-14.59%	-25.90%	-5.83%	-36.17%	-45.51%	-6.08%	-14.91%	-21.18%	-6.47%	-28.39%			
Skew	30.13%	190.08%	771.21%	-38.48%	-122.82%	18.02%	-18.61%	-463.77%	117.75%	-22.72%	444.24%	13.01%	-16.04%			
Semi Variance	1.20%	1.55%	1.35%	1.97%	2.61%	1.51%	4.00%	3.78%	1.15%	1.75%	1.97%	1.47%	3.32%			
VAR	-3.20%	-3.33%	-3.26%	-3.95%	-2.70%	-4.24%	-4.10%	-2.88%	-2.65%	-3.48%	-2.77%	-3.79%	-3.23%			
1395																
	beterans	petrol	tapico	hekeshti	khepars	khodro	shebandar	shapna	fazar	foolad	kachad	hamrah	vaansar	vabank	vaghadir	vamaaden
Average	-0.03%	0.01%	-0.06%	-0.16%	-0.22%	-0.22%	-0.02%	-0.05%	-0.20%	0.00%	-0.04%	0.06%	-0.04%	0.01%	-0.10%	0.01%
Variance	0.04%	0.10%	0.01%	0.01%	0.06%	0.06%	0.03%	0.03%	0.04%	0.02%	0.03%	0.00%	0.01%	0.01%	0.01%	0.04%
Standard Dev	1.89%	3.16%	1.13%	1.08%	2.37%	2.35%	1.85%	1.76%	1.90%	1.51%	1.66%	0.69%	1.08%	1.14%	1.05%	2.05%
Kurt	108.80%	15790.70%	1376.69%	1594.84%	-45.58%	-18.42%	69.20%	124.83%	31.30%	141.60%	271.13%	826.75%	678.50%	538.36%	299.35%	416.66%
Max	4.75%	44.28%	5.59%	4.50%	4.80%	5.01%	4.73%	4.61%	4.82%	4.79%	5.40%	4.26%	4.51%	4.30%	4.66%	11.66%
Min	-5.11%	-4.19%	-7.90%	-8.27%	-5.06%	-5.13%	-4.72%	-4.96%	-4.95%	-4.22%	-4.98%	-2.75%	-4.57%	-4.40%	-3.62%	-4.55%
Skew	30.79%	1129.57%	-34.75%	-128.28%	13.63%	13.73%	4.45%	33.94%	48.15%	41.48%	56.91%	125.69%	125.97%	107.78%	72.46%	119.49%
Semi Variance	1.28%	0.85%	0.94%	0.93%	1.36%	1.43%	1.30%	1.11%	1.04%	0.94%	1.05%	0.39%	0.67%	0.68%	0.63%	1.08%
VAR	-3.39%	-2.02%	-1.35%	-1.23%	-4.21%	-4.55%	-3.40%	-2.64%	-2.95%	-2.43%	-2.44%	-0.76%	-1.56%	-1.43%	-1.67%	-3.04%

4. Results

In this research for numerical solving, the financial information of fifty companies were extracted daily for 2012 to 2017 then we optimized them by helping 3 algorithm i.e. black hole, gravity Research and hybrid, which are explained in the following section. Also the number of selected companies in portfolio is equal to 8. For using the provided methods in this research, we can use annual or daily return if there is time series sufficiently for calculation covariance matrices in case of required and the expected return estimate

After end of these stages, we use black hole, Gravitational Research and hybrid for solving the problem.

Namely, it is solved with selection 50 stocks. Figures 1, 2 showing the obtained efficient frontier from the developed model along with the obtained information from the assigned percent to selected stocks in each amount the past performance index of portfolio that these results were obtained by using applying the written code for these compounds in MATLAB software

While for each operating load, the amount code R_d changes in limitation that this amount has fluctuation between maximum and minimum returns which between 51 given amount R_d to written code, each time operating the number of them place on the Efficient frontier in the optimized region.

Since we follow the most optimized state in portfolio in operating algorithm, the amounts that are shown like zero in each point of Efficient frontier for investor have high risk and or less return due to the assigned percent to each stock is zero. So the written code shows the most optimized possible sate for us each point on the efficient frontier.

In the following, the Efficient frontier figures are shown in model.

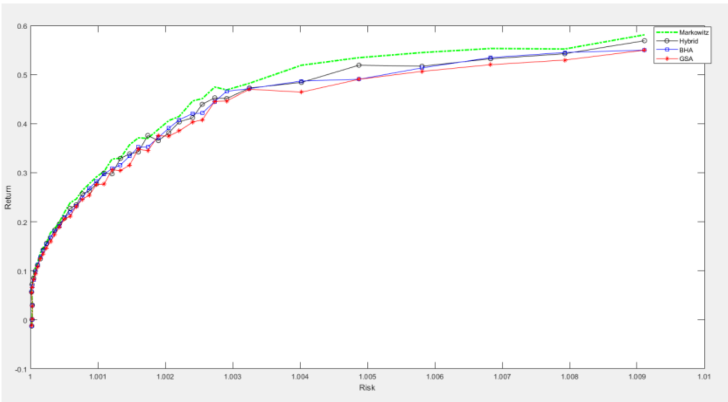

Figure 1: The obtained Efficient frontier from algorithms

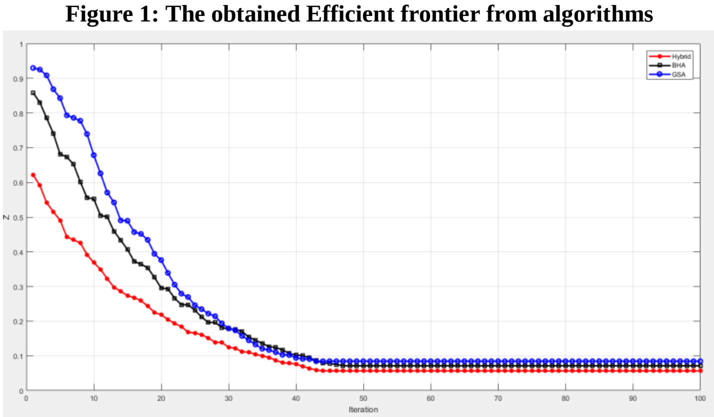

Figure 2: the obtained convergence figure from algorithm

As it is clear from figure, the algorithm status is similar firstly and finally, the greater the risk, the difference between these three algorithms is revealed. In this status, hybrid algorithm has the better results.

As the above results show that, this sample for model can do the optimization, as expected here, the Efficient frontier is discontinuous due to search space discontinuous and limitations effects. Companies whose have positive return during their life have obtained the most amount of stock from portfolio. In all stages, the predicted models can optimize the issue in the region and even when limitation allow, find the points on main efficient frontier. This affair is showing hybrid algorithm power at solving portfolio issues.

The above figures show that portfolio Efficient frontier have concavity and indentation. In fact, this figure show that the above model is efficient and can overcome on the computed challenges drawing the efficient frontier.

4.1 The numerical solving daily and annually during 2012 to 2017

For better comparison, we considered the data suggested methods daily and we considered them annually for 2012 to 2017 and the results are explain in the following.

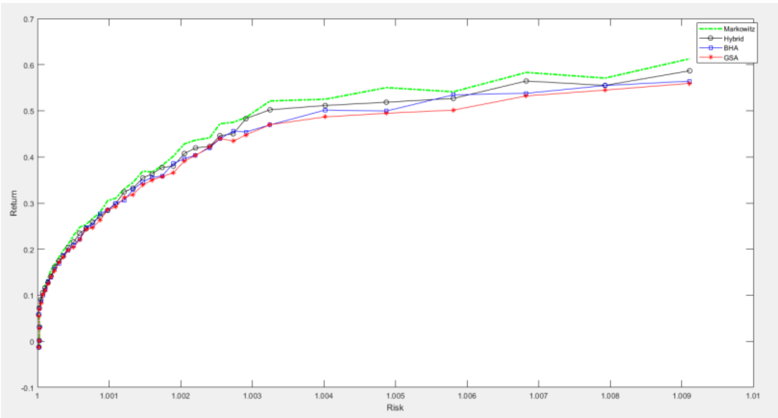

Figure 3: the obtained Efficient frontier from algorithms for 2012

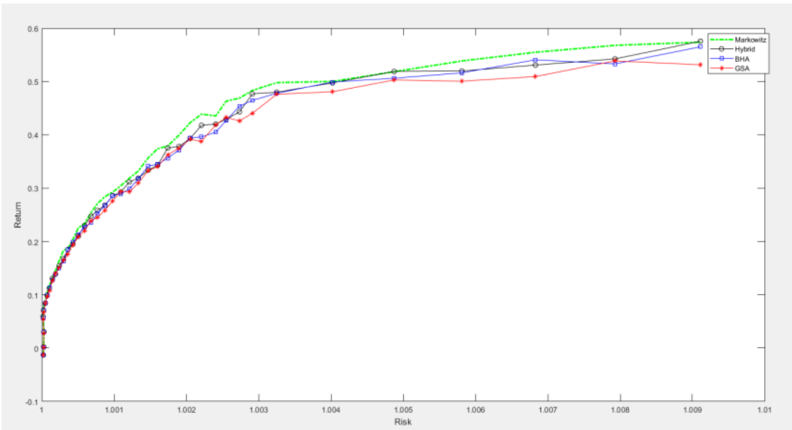

Figure 4: the obtained Efficient frontier from algorithms for 2013

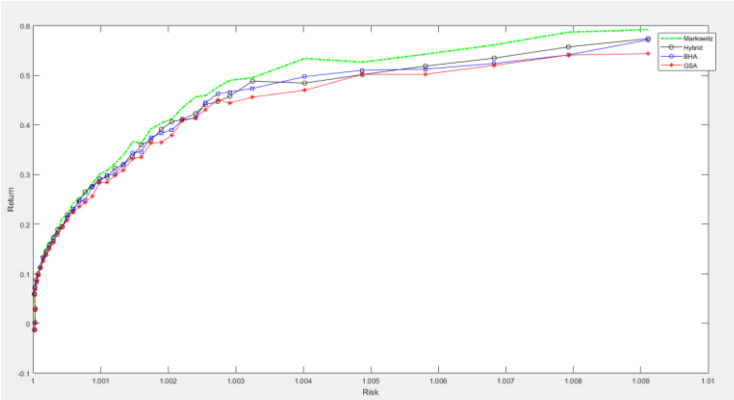

Figure 5: the obtained Efficient frontier from algorithms for 2014

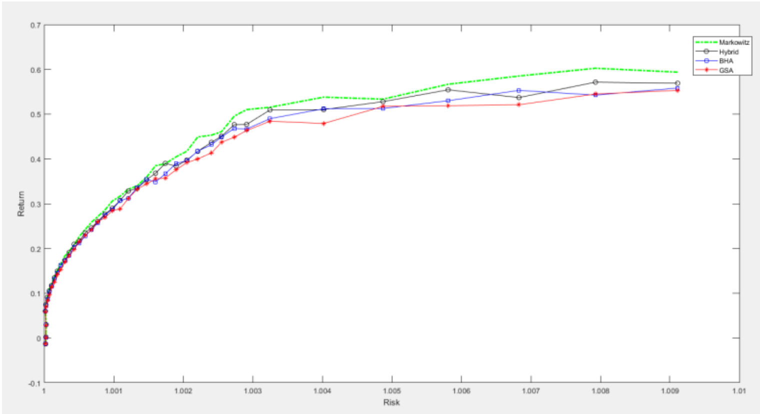

Figure 6: the obtained Efficient frontier from algorithms for 2015

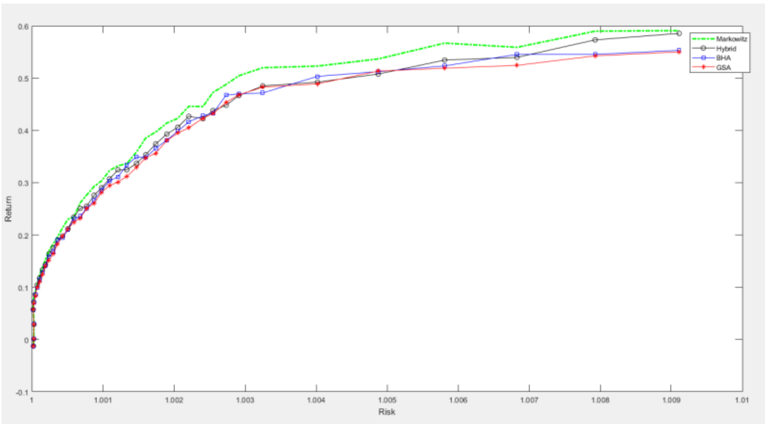

Figure 7: the obtained Efficient frontier from algorithms for 2016

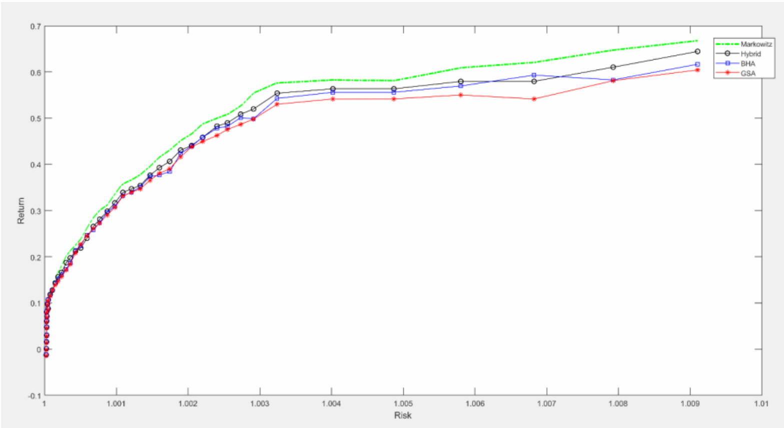

Figure 8: the obtained Efficient frontier from algorithms for 2017

For comparing Algorithms, we test whether current distances standard deviation squares errors between Efficient frontier metaheuristic models with markowitz model is there significant different or not?

The error amount of standard deviation squares are obtained from the following formula:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(Model - Markowitz)^2}{N}}$$

Table 2: Comparing algorithms distance with markowitz model

RMSE amount algorithms with markowitz model			
Year	**Hybrid algorithm**	**Black hole algorithm**	**Gravitational Research algorithm**
2012	0.010144	0.023817	0.024012
2013	0.017632	0.024191	0.024214
2014	0.010028	0.021102	0.021816
2015	0.014589	0.022056	0.022998
2016	0.020472	0.0237261	0.024109
2017	0.017564	0.023847	0.024302

As it is clear from the results that in all the years, the introduce hybrid method has obtained the closest answer to exact answer in this research that is Markowitz but the calculated disputations in the above table require to the exact statistic comparison until can state whether there is significant different among Hybrid algorithms RMSE amounts, black hole, gravitational Research with each other and with Markowitz model or not?

For getting answer, this question is necessary, that is used from suitable statistics tests. Following the WALD test is examined for examination the existence the significant difference among Hybrid algorithms RMSE amounts, black hole, Gravitational Research ith each other and with Markowitz model and the determination coefficient for comparing explained power of these models

In order to exact calculation return and specially investments risk, the different models are created for prediction risk until the amount of risk is calculated regarding to model need and manner data scattering. The significant different quality and adjusted explained power of three models is examined by helping WALD test. Following, WALD test statistics is compared for new models for independent variables. Null hypothesis and contrast hypothesis are written for WALD test as follows:

Firstly, for comparing black hole and Markowitz models

H0: optimization the optimized portfolio of Markowitz model and black hole metaheuristic model has not similarity.

H1: optimization the optimized portfolio of Markowitz model and black hole metaheuristic model has similarity.

Then, for comparing hybrid method and Markowitz

H0: optimization the optimized portfolio of Markowitz model, and black hole metaheuristic model has not similarity.

H1: optimization the optimized portfolio of Markowitz model, and black hole metaheuristic model has similarity.

Table 3: WALD test for under examined models

Model No	WALD statistics	Significant level	Result
Markowitz	4.42	0.000	Coefficient significant confirmation
Hybrid	5.12	0.000	Coefficient significant confirmation
Black hole	4.64	0.000	Coefficient significant confirmation
Gravitational Reseach	3.23	0.000	Coefficient significant confirmation

It is necessary to note that WALD statistics compare algorithms by using F Fisher statistic distribution. As it is clear that the null hypothesis is rejected and can state that optimization the optimized portfolio of Markowitz and black hole metaheuristics model has similarity. Also optimization the optimized portfolio of Markowitz model has similarity with hybrid method. Also, in all years the introduced hybrid method in this research is the closest answer to exact answer i.e. Markowitz are obtained.

Results examination

Statistics hypothesis 1-1

H0: there is no significant difference between optimized portfolio Efficient frontier of Markowitz model and black hole metaheuristic model

H1: there is significant difference between optimized portfolio Efficient frontier of Markowitz model and black hole metaheuristic model

According to the obtained result, H0 hypothesis was confirmed.

Statistics hypothesis 1-2

H0: there is no significant difference between optimized portfolio Efficient frontier of Markowitz model and Gravitational Research metaheuristic model

H1: there is significant difference between optimized portfolio Efficient frontier of Markowitz model and Gravitational Research metaheuristic model

According to the obtained result, H0 hypothesis was confirmed.

Statistics hypothesis 1-3

H0: there is no significant difference between optimized portfolio Efficient frontier of Markowitz model and hybrid metaheuristic model

H1: there is significant difference between optimized portfolio Efficient frontier of Markowitz model and hybrid metaheuristic model

According to the obtained result, H0 hypothesis was confirmed.

Comparing the explained power

The determination coefficient, that is called recognizing coefficient, shows that how many percent changes dependent variable is explained by independent variables or in other words, the other determination coefficient showing that" how much dependent variable changes under effect independent variable and the other changes dependent variable related to other factors. We examine comparison explained power to comparison adjusted determination coefficient in regression equations.

Table 4: The comparison of models adjusted determination coefficient

The desired model	Determination coefficient	The adjusted determination coefficient	Rank
Markowitz	0.40	0.34	3
Hybrid	0.47	0.43	1
Black hole	0.44	0.41	2

The important difference of determination coefficient and the adjusted determination coefficient is that determination coefficient assumes that each observed independent variable in model; explain the current changes in dependent variable, therefore, the observed percent by determination coefficient is by the assumption of the effect of all independent variables on dependent variable. If the observed percent by the adjusted determination coefficient only is resulted from the real affects model independent variables on dependent and not all dependent variable. The other difference is that the suitable variables for the model by determination even with high amount are not recognizable if we can trust the adjusted determination coefficient estimated amount. We observe in the above table that the explained power of hybrid method is more than other and Markowitz is less than other and the black hole is the model that place between two models in terms of the explained power.

5. Discussion and Conclusions

In the comparison of this research with past research that mentioned to the most important of them in review literature, the current research is the first research that get the efficient portfolio, they use from black hole, Gravitational Research and hybrid algorithm, moreover; in order to test the algorithm efficiency, they are compared with with the markowitz model as well as their distance with the obtained Efficient frontier are obtained from the Markowitz model. Using the above efficiency test, in exact manner the used algoritm capability amount for solving issues criteria appear. Despite, the research need to algorithms capability proof in sloving the critteria problems, we can find less researches in Iran security that use the above method for proofing his algorithm capability. In addition to the mentioned advantages, the Efficient frontier distance obtained black hole, Gravitational Research and hybrid algorithms is computed with Efficient frontier Markowitz model that has been satisfied with visual inspection in most studies; moreover, although the feature of used computer systems is different in various articles but the examinaton of case study results show that hybrid, black hole, Gravitational Research algorithm has high speed for sloving the problems of optomized portfoloio.

The other innovation should be examined the current research in the new metaheuristic algorithm. Since the black hole metaheuristic algorithm, the newly innovated Gravitational Research , the current research can consider in the first researches that is used. Also for the first time, this two algorithm was combined until they obtained the more optimized result than two algorithm.

The aim of this research was to show that we can use from black hole metaheuristic, Gravitational Research , and combined (hybrid) algorithms for portfolio optimization rather than Markowitz model with higher speed and accuracy for optimization portfolio. So we examine the subsample of the conducted similar researches with other metaheuristic algorithms in the field of optimized portfollio.

In the 2011, Raei and their colleague use from harmony search algorithm for optimization portfolio in Iran security market (Raei and Ali Beigi 2011). In this study, semi-variance-avregae approach is used and the portfolio Efficient frontier are obtained with finding 50 points by algorithm.

In other research that Raie and their colleague conducted in 2011, the portfolio Efficient frontier was obtained with optimized algorithm of the particle cumulative movement (Raei et.al, 1389). Algorithm is operated on Iran stock exchange and do the Efficient frontier in different modes. The result of the reseach also show that algorithm for searching efficeint frontier at proper time, have the well accuracy.

The many researches are done for using metaheuristic algorithms. The former researches have shown that metaheuristic algorithms can with suitable accuracy than math exact solution, solve its problem (Cheng et.al ,2000).

Marinaky and their colleague in the research under title " the bee optimization algorithm for financial problems classification", the bee algorithm for selection the usage financial suitable variables features and then they used it for financial problems classifictions, researchers the bee algorithm results with the obtained results from particle cumulative movement algorithm and Ant Colony Algorithm, result of this comparison showing the high performance bee algorithm

Regarding the findings of this reserarch, it is sugested that investor for sloving portfolio selection problems, due to accpeted speed solving model, they use from Gravitational Research , black hole, hybrid algorithms

if efficiency and speed, both of them have important for selection portfolio, it is sugested that use hybrid algorithm for portfolio selection because according to the obtained results from this research, the higher efficiency hybrid algorithm was proofed than black hole and Gravitational Research algorithm.

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