Enhanced indexing using a discrete Markov chain model and mixed conditional value-at-risk

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ABSTRACT

Enhanced indexing (EI) is a passive investment strategy that seeks to perform better than the benchmark index in the sense of higher return. The purpose of enhanced indexing is to determine optimal portfolios with the maximum excess mean return over the index return. The less efficient markets offer scope for enhanced indexing. The less (more) efficient the market is, the greater (lesser) is the chance of beating it. In this study, a two-step procedure is proposed for enhanced indexing of the Tehran Exchange Dividend and Price Index (TEDPIX). In the first step, a discrete Markov chain model is designed to filter stocks based on their high probability of gain over the benchmark index. In the second step, optimal weights are assigned to the filtered assets by maximizing the STARR ratio with MCVaR. The sample includes weekly data from March 2013 to March 2020. The data is divided into a 26-time frame, including 52 in-sample data and 12 out-of-sample data. The results of 26 window (containing a rolling data set of 52 weeks in- sample data & 12 weeks out-of-sample) show that not only the portfolio return positively correlated to the TEDPIX return and could track it entirely, but also it could exceed and enhance the portfolio tracking. More precisely, our model portfolio could grow 13.65 times while the TEDPIX grows just 6.5 times simultaneously.

Keywords:

Discrete Markov chain, Enhanced indexing, Mixed conditional value-at-risk, Portfolio optimization, STARR ratio.

1. Introduction

In finance, the expression, *index funds,* identifies management strategies that have the objective of tracking the performance of a specific market index (the so-called *benchmark*), attempting to match, as much as possible, its returns. This investment strategy, usually called indexing or *index tracking,* is a passive form of fund management where the manager has a low degree of flexibility, and the fund is expected to reproduce the performance of the benchmark by adequately choosing a representative selection of securities. The index tracking problem aims at minimizing a function, called the *tracking error*, which measures how closely the portfolio mimics the performance of the benchmark.

In contrast, the term *enhanced indexing* refers to an investment strategy that, while still attempting to track the market index, is specifically designed to find a portfolio that outperforms the benchmark. In other words, the manager of an enhanced index fund is trying to achieve a higher return than the benchmark but incurring a minimal additional risk, as measured by the tracking error. Riepe and Werner (1998) comment that "defining enhanced indexing is difficult because so many purveyors of enhanced index products describe their mission differently (Guastaroba et al., 2020).

In practice, fund managers use several techniques to increase the return above that of their targeted index. Grossman and Algert (1998) and Riepe and Werner (1998) discuss the strategies. These strategies are classified into the following categories.

1) *Security selection*. Most enhanced index fund managers use traditional fundamental and technical analysis to select stocks. This process is precisely the same as that of a typical active fund.
2) *Yield curve enhancement*. Funds can obtain index exposure by purchasing futures and investing the cash portion of their portfolio in higher-yielding, fixed income securities. The alpha can be increased by taking on additional duration risk (moving up the yield curve) or taking on credit risk. The risks in this strategy come from unanticipated shifts in the yield curve, credit risk, and potentially overpaying for futures contracts at the time the manager needs to roll over the position.
3) *Equity market neutral*. This index exposure is obtained by purchasing index futures. The alpha is provided by a market-neutral long-short portfolio. The long-short part of the portfolio is typically managed using quantitative tools.
4) *Derivatives-based and leveraged strategies*. These strategies combine a futures-based indexing strategy overlaid with an options portfolio. Derivatives also could be used to achieve a beta greater than 1.

French (2008) reports that active management strategies do not perform better on average than the market. So, it is not surprising that, according to the Moody's report (2017), almost one-third of all investments in the United States, or approximately $6 trillion, are in index funds or other passive management strategies.

In recent years, index funds and enhanced index funds have received more attention. While the best of the actively managed funds outperforms the market in any particular year, over the long-term, the majority of such funds do not (e.g., in the U.K. in 1998, only one-quarter of actively managed funds outperformed their comparative index over a five-year period). Also, an actively managed fund that outperforms the market one year may fail to do so in subsequent years (e.g., in the U.K., many funds that performed well in 1992 had fallen to bottom quartile positions by 1998). Moreover, as stock markets (and so their indices) have historically risen in the long-term, it has become clear that reasonable returns can be obtained without incurring the additional risks associated with active management (Beasley, Meade, and Chang 2003).

In order to study the situation of actively managed mutual funds in the Iran capital market, Eyvazlu et al. (2021) examined the average return of these funds for ten years. The comparison of the monthly average return of actively managed mutual funds with the monthly return of the Tehran Exchange Dividend and Price Index (TEDPIX) from March 2010 to early March 2009 shows that the TEDPIX in-sample period has outperformed the average of actively managed mutual funds' return. The monthly average return of actively managed mutual funds during the mentioned period was equal to 2.66 percent, while the monthly average return of the TEDPIX during the same period was equal to 2.89 percent. Also, as shown in Figure 1, the TEDPIX with a cumulative monthly return performance of 2.067 percent outperformed the average of actively managed mutual funds during the studied period. Note that during the same period, the average cumulative return of actively managed mutual funds was 1.601%. Thus, actively managed mutual funds, on average, had almost worse performance than the TEDPIX. Therefore, passive investment strategies such as index tracking, as well as enhanced indexing, can be considered as an alternative approach.

Finally, considering the importance and undeniable role of index tracking and enhanced indexing in the prosperity of capital markets, the study and implementation of the recent approach are on the list of this research. The purpose of this study is to investigate a mathematical optimization model, which has a performance beyond the index by selecting a limited number of stocks of an index. So, for this study in the first step, a discrete Markov chain model has been designed to filter a few stocks based on their high probability of gain over the benchmark index. In phase two, optimal weights to the filtered assets have been assigned through maximizing the STARR ratio with MCVaR.

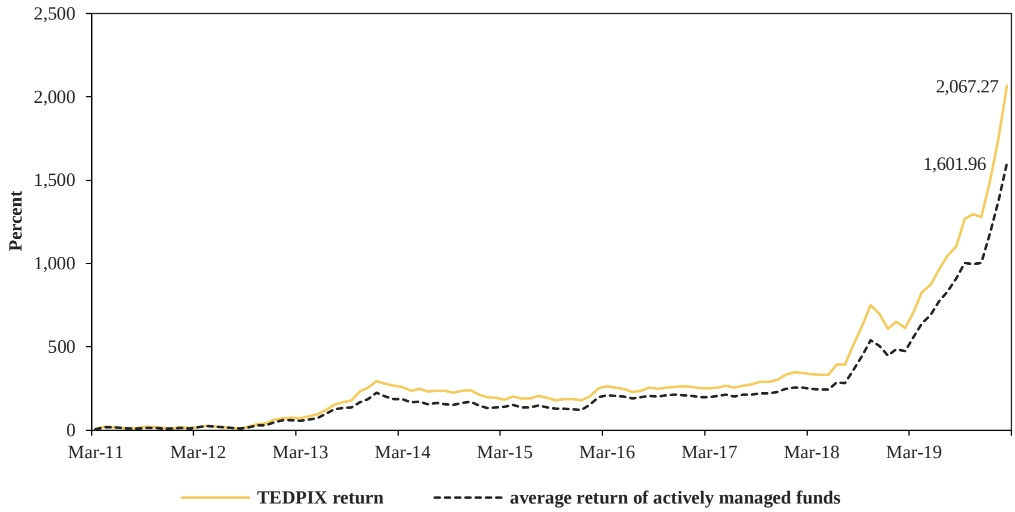

Figure 1. Comparison of the average return of actively managed funds with the index return of the Tehran Stock Exchange during the period from the beginning of March 2011 to the beginning of March 2020.

2. Literature review

Recent researches show that enhanced index funds have received more attention. The Goel et al. (2018) paper can be referred to for the literature review. Ahmed and Nanda (2005) report the growth of the enhanced index funds over 20 years, where most of such funds target the S&P 500 index. Koshizuka et al. (2009) and Weng and Wang (2017) report the increasing popularity of the enhanced index funds in Tokyo and Chinese markets, respectively. The findings of Weng and Wang (2017) further suggest that passive index funds are more beneficial than the active funds in the Chinese market.

Wu et al. (2007) use the goal programming approach for E.I. by setting the desired goal on portfolio excess mean return and the tracking error. Koshizuka et al. (2009) propose an optimization model using mean absolute deviation (MAD) and Semi MAD to find an index-plus-alpha portfolio[1] among all those portfolios which are positively correlated to the index. Canakgoz and Beasley (2009) propose a bi-objective mixed linear E.I. model where the alpha of the portfolio is maximized, and the beta of the portfolio is minimized around unity. Li et al. (2011) formulate the bi-objective programming problem of maximizing the excess mean return and minimizing the downside deviation of order two from index return with transaction cost constraints. An immune-based multi-objective optimization algorithm is employed to find a solution to the problem. Lejeune (2012) proposes to maximize the excess mean return of the portfolio under the bound on the market risk quantified by the semi-deviation measure. He provides the game theoretical framework when the distribution is known to belong to the ellipsoidal distribution family. Guastaroba and Speranza (2012) form a mixed-integer linear program for I.T. by minimizing the absolute deviation between portfolio return and index return to track the index while imposing constraints on cardinality and transaction cost. They also extend this model to E.I. by tracking the index-plus-alpha portfolio. They find the solution for the I.T. problem under the kernel search framework. Filippi et al. (2016) apply a heuristic approach which combines the kernel search with the ϵ-constraint method to solve the integer linear bi-objective programming problem of maximizing the excess mean return and minimizing the absolute deviation between returns of portfolio and index with real-life features.

Bruni et al. (2015) propose a linear program for E.I. by maximizing excess mean return and impose a bound on the worst performance of portfolio return from the index. Paulo et al. (2016) propose an optimization model that creates a trade-off between the weighted sum of excess mean return and variance of the difference of returns of portfolio and index. They allow short-selling and hence obtain the close form solution for the optimal weights when the assets for investment are prior selected. Guastaroba et al. (2016) form a linear program optimizing the Omega ratio for E.I. with fixed and random targets. They also extend their models to incorporate real features like transaction cost and cardinality constraint and obtain a mixed-integer program. Later, in the same year, Guastaroba et al. (2016) optimize the Omega ratio under the risk-reward framework to propose a new optimization model for E.I. that uses the mixed CVaR.

For the first time in Iran, Hanifi et al. (2009) raised the issue of index tracking. Using the genetic algorithm in three approaches of classical, enhanced, and multi-stage genetic algorithm and considering the number of stocks allowed in the portfolio in four modes of 5, 10, 15, and 20 stocks, they solved the index-tracking portfolio problem. Their results showed that the multi-stage genetic algorithm had the lowest tracking error, among all other approaches. Varsehee and Shams (2010) presented an innovative solution method to form an index tracking portfolio. Constraints used in the model included an integer limit on the number of stocks allowed in the portfolio, as well as an investment amount ceiling and floor. They divided the issue into two sub-issues: stock selection and optimal weight assignment. Their solution was based on a reduction in the study scope using the concept of robust correlation. Their sample contains 30 T.S.E. listed stocks, and they solved the problem in a definite method. Nabizadeh et al. (2017), using two evolutionary genetic algorithms and differential evolution algorithm, investigated the performance of the three models presented in their research. After evaluating the results, they find that the model based on an undesirable beta, which has been solved by a differential evolution algorithm, is much more efficient. They solved the problem by considering the minimum number of stocks in the portfolio, regardless of transaction costs. Eyvazlu et al. (2017) also evaluated the use of co-integration and correlation in the formation of an index-based stock portfolio by examining the overall index and showed that due to the tracking error, the co-integration approach outperforms the correlation approach. On the other hand, based on portfolio returns, information ratio and, Sharp ratio, model performance in index-tracking is better than the enhanced indexing. Ansari et al. (2020) used a two-stage model based on integrated integer programming to minimize tracking error and maximize returns under tolerance values for tracking error. They used the index of 50 active stock exchange companies to show the efficiency of the proposed model. The research findings show that the proposed two-stage model is better than the one-stage model.

3. Methodology

3.1. Notations

Similar to the study of Goel et al. (2018), Consider a portfolio P of n-assets $P = \{w_1, \dots, w_n\}$, where w_i is a decision variable denoting the proportion of total budget to be allocated to the ith asset, $i = 1, \dots, n$. Let the investment horizon be Γ. It is customary to partition Γ into an equal number of time points, say T, to observe the jth realization (a particular value), $j = 1, \dots, T$, of each asset. Let r_{ij} denote the jth realization of return of the ith asset with probability p_j with mean value $\mu_i = \sum_{j=1}^{T} r_{ij} p_j, i = 1, \dots, n$. The jth realization of return for the portfolio P is $R_{j(w)} = \sum_{i=1}^{n} r_{ij} w_i$, with probability $p_j, j = 1, \dots, T$. In this way, the return of the portfolio P, denoted by $R(w)$, is finitely distributed $\{R_1(w),\ldots,R_T(w)\}$ with corresponding probability vector $\{p_1,\ldots,p_T\}$.

3.2. Tail risk measures

Value-at-Risk (VaR_α) measures the maximum possible loss at a confidence level $\alpha \in (0,1)$, and it is defined as follow:

$$VaR_\alpha(-R(w)) = \min\{r \in \mathbb{R} \mid F_{-R(w)}(r) = \Pr(-R(w) \le r) \ge \alpha\} \tag{1}$$

where $F_{-R(w)}(r)$ is the distribution for the portfolio loss $-R(w)$. If the portfolio returns are assumed to be normally distributed, then the portfolio optimization model minimizing VaR_α risk measure is a convex program and hence has a global solution (Linsmeier and Pearson 1996).

Conditional Value-at-Risk ($CVaR\_\alpha$) is the mean of the following α-tail distribution of $-R(w)$ described by Rockafellar and Uryasev (2002):

$$G_\alpha(-R(w),r) = \begin{cases} 0, & r < VaR_\alpha(-R(w)) \\ \dfrac{F_{(-R(w))}(r)-\alpha}{1-\alpha}, & r \ge VaR_\alpha(-R(w)) \end{cases} \tag{2}$$

The pioneering work of Rockafellar and Uryasev (2000) establishes that the minimization of $CVaR_\alpha$can be approximated by a linear program for the case of continuous distributions through sampling techniques, like, Monte-Carlo simulation. The $mean-CVaR_\alpha$optimization problem is given as follows:

$$(CVaR_\alpha) \qquad min\,\beta + \frac{1}{(1-\alpha)}\sum_{j=1}^{T} p_j u_j \tag{3}$$

$$\begin{aligned} &subject\ to \\ &u_j + \beta + \sum_{i=1}^{n} r_{ij} w_i \ge 0, j = 1,\ldots,T \\ &\sum_{i=1}^{n} \mu_i w_i \ge R^\star \\ &w \in X,\ \ u_j \ge 0, j = 1,\ldots,T \end{aligned}$$

where $X = \{(w_1,\ldots,w_n)' : \sum_{i=1}^{n} w_i = 1, 0 \le w_i \le 1, i = 1,\ldots,n\}$ is the set of all feasible portfolios with no short selling and the budget constraint, $u_j = \left(-\sum_{i=1}^{n} r_{ij} w_i - \beta\right)^+, j = 1,\ldots,T,$ are T, auxiliary variables, and R^* is the minimum threshold preset by an investor on the expected return from the portfolio.

The ($CVaR_\alpha$) model attempts to minimize the expected value of the left tail of the return distribution of the portfolio. Often, due to inaccurate estimation of this tail, the optimal portfolio from ($CVaR_\alpha$) model fails to perform well in the actual investment period (or out-of-sample period). It is therefore desired to consider other parts of the return distribution for more informed decision making. The mixed $CVaR$ is a generalized version of $CVaR$, which extracts more information from the return distribution by including $CVaR_\alpha$ for different values of α and aggregating them by attaching relative importance to each of them (Goel et al., 2018).

3.3. Mixed Conditional Value-at-Risk

Mixed $CVaR$ is a weighted sum of multiples $CVaR$ risk measures at different confidence levels α. For m distinct values α_k, $k = 1,\ldots,m$, with $0 \le \alpha_m \le \cdots \le \alpha_1 < 1$, the $MCVaR$ is defined as follows (Goel et al. 2018):

$$MCVaR(-R(w)) = \lambda_1 CVaR_{\alpha_1}(-R(w)) + \cdots + \lambda_m CVaR_{\alpha_m}(-R(w)) \tag{4}$$

where $\lambda_k > 0$, $k = 1,\ldots,m$, and $\sum_{k=1}^{m} \lambda_k = 1$. For $\alpha_m = 0$, $CVaR_{\alpha_m}(-R(w))$ is the mean of the random variable $-R(w)$. The set Λ is defined as follows:

$$\Lambda = \left\{\lambda = (\lambda_1,\ldots,\lambda_m)' : \lambda_k \ge 0, k = 1,\ldots,m, \sum_{k=1}^{m} \lambda_k = 1\right\}. \tag{5}$$

All good properties of $CVaR$, like coherent risk measure, SSD consistency, and linear mean-risk model, remain to preserve under $MCVaR$ (Mansini et al. 2007). Similar to the $CVaR$ minimization problem, minimizing $MCVaR$, with $\alpha_k \in (0,1)$, $k = 1,\dots,m$, and $\lambda \in \Lambda$, is a linear program and is given as follows (Goel et al. 2018):

$$(MCVaR_\alpha) \qquad \min_{\beta,w} \sum_{k=1}^{m} \lambda_k \left(\beta_k + \frac{1}{(1-\alpha_k)} \sum_{j=1}^{T} u_{jk} p_j \right) \tag{6}$$

$$\begin{aligned} & subject\ to \\ & u_{jk} + \beta_k + \sum_{i=1}^{n} r_{ij} w_i \geq 0, k = 1, \dots, m, j = 1, \dots, T \\ & u_{jk} \geq 0, k = 1, \dots, m, j = 1, \dots, T \\ & w \in X \end{aligned}$$

where $u_{jk} = \left(-\sum_{i=1}^{n} r_{ij} w_i - \beta_k\right)^+$, $k = 1, \dots, m$, $j = 1, \dots, T$, are auxiliary variables.

Mansini et al. (2007) describe the set of weights associated with the $MCVaR$ as follows:

$$\lambda_1 = (1-\alpha_2)(1-\alpha_1)/(1-\alpha_m)^2$$

$$\lambda_k = \big((1-\alpha_{k+1}) - (1-\alpha_{k-1})\big)(1-\alpha_k)/(1-\alpha_m)^2 \in \{2, \dots, m-1\} \tag{7}$$

$$\lambda_m = \big((1-\alpha_m) - (1-\alpha_{m-1})\big)(1-\alpha_m)/(1-\alpha_m)^2$$

with $0 \leq \alpha_m < \cdots < \alpha_1 < 1$.

3.4. A two-step procedure for enhanced indexing

In lines with the work of Goel et al. (2018), in step one, a strategy to filter some assets out of all assets constituting an index is designed. In step 2, an enhanced indexing optimization model is proposed to assign optimal weights to the filtered assets.

3.4.1. Step-1: Discrete Markov chain model for filtration criterion

Similar to the study of Goel et al. (2018), assets that have a high probability of yielding returns more significant than that of the index and their returns do not fall below a certain threshold are looked out. The difference of asset returns and index returns is modeled as a Markov chain with three states representing a good, neutral, and bad performance of an asset over the index. Next, the transition probability matrix and the stationary probability distribution are obtained for each asset. Based on the difference of the steady-state probabilities of good performance and bad performance of an asset over the index, a pre-determined number of assets is selected that having the maximum value of this difference.

Let I_j, r_{ij} denote the jth realization of returns of the index and the ith asset respectively, and $\epsilon_1 > 0, \epsilon_2 > 0$ denote the predefined threshold levels.

For each asset i, $i = 1, \dots, n$, an indicator process $Y_i = \{Y_{ij}, j = 1, \dots, T\}$ are defined as follows:

$$Y_{ij} = \begin{cases} 0 & -\epsilon_2 \leq r_{ij} - I_j \leq \epsilon_1 \\ 1 & I_j - r_{ij} > \epsilon_2 \\ 2 & r_{ij} - I_j > \epsilon_1 \end{cases} \tag{8}$$

This is a homogeneous discrete-time Markov chain with state-space $\{0, 1, 2\}$.

Using the historical data of ith asset returns and index returns, we construct the frequency transition matrix $F_i = \left[f_{wz}^i\right]_{3\times3}$, where $f_{wz}^i, w, z \in \{0,1,2\}$, denotes the transition frequency from state w to state z.

$$F_i = \begin{pmatrix} f_{00}^i & f_{01}^i & f_{02}^i \\ f_{10}^i & f_{11}^i & f_{12}^i \\ f_{20}^i & f_{21}^i & f_{22}^i \end{pmatrix} \tag{9}$$

Determine the one-step transition probability matrix $P_i = \left[p_{wz}^i\right]_{3\times3}$ as follows:

$$P_i = \begin{pmatrix} p_{00}^i & p_{01}^i & p_{02}^i \\ p_{10}^i & p_{11}^i & p_{12}^i \\ p_{20}^i & p_{21}^i & p_{22}^i \end{pmatrix} \tag{10}$$

with entries

$$p^i_{wz} = \begin{cases} \dfrac{f^i_{wz}}{\sum_{z=0}^{2} f^i_{wz}}, & \sum_{z=0}^{2} f_{wz} \neq 0 \\ \dfrac{1}{3}, & otherwise. \end{cases} \tag{11}$$

The stationary distribution $v_i = (v^i_0, v^i_1, v^i_2)$ by solving the linear system $v_i = v_i P_i$. Since the Markov chain is finite, irreducible, and aperiodic, the stationary distribution vector v_i exists and is unique.

The values $v^i_2 - v^i_1$ are arranged in the descending order, and the pre-decided top $\tau\%$ of the assets is picked from it.

The higher value of $v^i_2 - v^i_1$ indicates the ith asset has a high probability of performing better and a lower probability of performing worse than the index (Goel et al., 2018).

3.4.2. Step-2: STARR ratio models for enhanced indexing

$STARR$ ratio uses the $CVaR$ measure to control extreme losses in the return distribution concerning the index. An optimization problem with $STARR$ ratio is a linear program and hence tractable. This motivated us to concentrate on the $STARR$ ratio for solving enhanced indexing problems (Martin et al. 2005).

The $STARR$ ratio at α confidence level is defined as follows:

$$STARR_\alpha(R(w)) = \frac{E(R(w)) - E(I)}{CVaR_\alpha(I - R(w))} \tag{12}$$

If the excess mean returns and the $CVaR$ of the corresponding loss series are of the same sign, then the $STARR$ ratio is coherent in the sense that its numerator and denominator are coherent risk measures (RACHEV et al. 2008).

Different variants of the $STARR$ ratio can be formulated to take into account more information of worst cases by replacing the $CVaR$ measure by the $MCVaR$ measure, two tail $CVaR$ measure, two tail $MCVaR$ measure, and deviation $MCVaR$ measure. $MSTARR$ is one of these measures the uses $MCVaR$ instead of $CVaR$ and define the following:

$$MSTARR_\alpha(R(w)) = \frac{E(R(w)) - E(I)}{MCVaR_\alpha(I - R(w))} \tag{13}$$

Since $MCVaR_\alpha(I - R(w))$ is coherent and convex risk measure, $MSTARR(R(w))$, is a coherent and quasi-concave ratio.

The optimization model for $MSTARR(R(w))$ at confidence level $\alpha_k \in (0,1), k = 1, \dots, m$, and weight vector $\lambda \in \Lambda$, defined as the following quasi-concave optimization problem:

$$(MSTARR) \qquad \max_{\beta,w} \frac{\sum_{i=1}^{n} \mu_i w_i - E(I)}{\sum_{k=1}^{m} \lambda_k CVaR_{\alpha_k}(I - R(w))} \tag{14}$$

$$subject\ to$$
$$\sum_{i=1}^{n} w_i = 1$$
$$w_i \geq 0, i = 1, \dots, n$$

Applying transformation defined by Charnes and Cooper (1962), the following convex programming problem is received:

$$(MSTARR) \qquad \max_{\beta,w} \sum_{i=1}^{n} \mu_i \tilde{w}_i - \gamma E(I) \tag{15}$$

$$subject\ to$$
$$\sum_{k=1}^{m} \lambda_k CvaR_{\alpha_k}(\gamma I - R(\tilde{w})) = 1$$
$$\sum_{i=1}^{n} \tilde{w}_i = \gamma$$
$$\tilde{w}_i \geq 0, i = 1, \dots, n$$

where $\gamma = \frac{1}{\sum_{k=1}^{m} \lambda_k CVaR_{\alpha_k}(I - R(w))} > 0$ is a homogenizing variable, $\tilde{w} = \gamma w$.

Using the auxiliary variables $\tilde{u}_{jk} = \max(-(\sum_{i=1}^{n} r_{ij}\tilde{w}_i - I_j) - \tilde{\beta}_k, 0),\ k = 1, \dots, m, j = 1, \dots, T$, the above model gets translated into the following linear program:

$$(MSTARR) \qquad \max_{\beta,w} \sum_{i=1}^{n} \mu_i \tilde{w}_i - \gamma E(I) \tag{16}$$

$$subject\ to$$
$$\sum_{k=1}^{m} \lambda_k \left[\tilde{\beta}_k + \frac{1}{(1-\alpha_k)T} \sum_{j=1}^{T} \tilde{u}_{jk} \right] = 1$$
$$\tilde{u}_{jk} + \left(\sum_{i=1}^{n} r_{ij}\tilde{w}_i - \gamma I_j \right) + \tilde{\beta}_k \geq 0,$$
$$k = 1, \dots, m, \qquad j = 1, \dots, T$$
$$\sum_{i=1}^{n} \tilde{w}_i = \gamma$$
$$\tilde{w}_i \geq 0, i = 1, \dots, n$$
$$\gamma > 0, \tilde{u}_{jk} \geq 0,\ k = 1, \dots, m,\ j = 1, \dots, T.$$

3.5. Sample data

In this study, the target index is the Tehran Exchange Dividend and Price Index (TEDPIX). Also, the sample includes weekly data from March 2013 to early March 2020 of all assets constituting an index.

3.6. Data analysis method

In lines with the earlier work of Bruni et al. (2017), a rolling window scheme of 12 weeks period is followed. The in-sample periods (training) and the out-of-sample periods (testing) are of 52 weeks and 12 weeks, respectively. By sliding the in-sample period by 12 weeks, a total of 26 windows for the data set is gotten (Figure 2.)

The weekly returns are calculated using $r_{ij} = \frac{P_{ij}-P_{ij-1}}{P_{ij-1}}, i = 1, \dots, n, j = 1, \dots, T$, where P_{ij} and P_{ij-1} are respectively the closing prices of the ith asset in jth and $(j-1)$th week.

According to the requirements of the research, $\epsilon_1 = \epsilon_2 = 0.02$; $\tau = 25$; $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\} = \{0.99, 0.95, 0.9, 0.85, 0.8\}$ is set. The weight vector λ is also computed using Eq. (7).

3.7. The research questions

1) Is there any significant correlation between research model portfolios returns and the TEDPIX returns?
2) Do the research model portfolios outperform the TEDPIX?

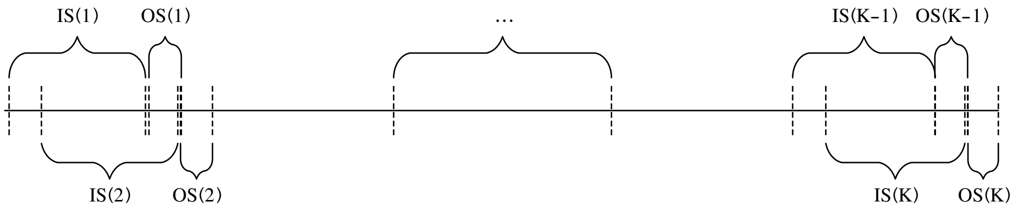

Figure 2. Window scheme for K periods; $\mathrm{IS}(k), k = 1, \dots, K$ indicates the in-sample data of kth window scheme, and $\mathrm{OS}(k), k = 1, \dots, K$ indicates the out-of-sample data of kth window scheme.

4. Results

Table 1 shows the performance of out-of-sample portfolios obtained from the enhanced indexing model in terms of correlation with the TEDPIX and excess mean return in 26 windows schemes. As the number of out-of-sample data is 12 weeks, in some observations, the correlation of the research model portfolios returns, and the TEDPIX returns is logically insignificant. More precisely, 23 out of 26 windows significantly correlated with 95% confidence level, and only two out of 26 windows have an insignificant correlation[2]. Also, 17 windows scheme correlations are significant at 99% confidence level.

For further scrutiny, we have tested the above results overall to have a definite conclusion. So, we have computed the correlation test on the total data, which was 0.8894. Thus, the portfolio returns and TEDPIX returns were positively-correlated at 99% confidence level.

Table 1 also indicates the average excess return of each. As presented, 19 out of 26 windows had positive excess returns, which shows the model has outperformed the TEDPIX in several times.

In table 2, we have represented the total performance of portfolios obtained from the enhanced indexing model on both the correlation between portfolios and TEDPIX and average excess return.

Table 1. The out-of-sample performance of the portfolios obtained from the research model on the correlation between portfolios and the TEDPIX and excess mean returns in each window scheme.

Window	Correlation	Excess mean return (%)
1	۰,۷۷۲***	-۰,۴۱
2	۰,۶۴۸**	۰,۵۴
3	۰,۸۹۲***	۰,۲۸
4	۰,۷۹۵***	۰,۴۴
5	۰,۹۱۹**	۰,۴۵
6	۰,۸۴۳***	-۰,۱۲
7	۰,۸۶۵***	۰,۳۷
8	۰,۹۳۴***	-۰,۰۷
9	۰,۶۶۵**	-۰,۳
10	۰,۵	۰,۷۹
11	۰,۳۵۶	۰,۴۹
12	۰,۵۷*	۰,۲۸
13	۰,۷۶***	۰,۱۲
14	۰,۶۹۹**	۰,۱۱
15	۰,۶۹۱**	-۰,۱۶
16	۰,۹۰۷***	۰,۰۸
17	۰,۸۸۳***	-۰,۲۸
18	۰,۸۱۳***	۰,۱۶
19	۰,۹۹۲***	۰,۳۱
20	۰,۹۸***	۰,۰۹
21	۰,۹۵۸***	۰,۲۷
22	۰,۹۵۶***	-۰,۰۶
23	۰,۸۳۵***	۱,۲۳
24	۰,۷۴۶**	۰,۰۷
25	۰,۷۵۱***	۰,۴۲
26	۰,۹۳۹***	۱,۲۵

The significance levels $0 < \alpha \leq 0.01$, $0.01 < \alpha \leq 0.05$, $0.05 < \alpha \leq 0.1$, *are displayed by ***, **, * respectively in the statistical test of correlation.*

Table 2. The total out-of-sample performance of the portfolios obtained from the research model on the correlation between portfolios and the TEDPIX and excess mean returns.

Correlation	Excess mean return (%)
0.8894***	0.2459

In order to test the second hypothesis, we used tail compare mean test on the out-sample in which $H_0: \mu_{P-I} = 0$ and $H_\alpha: \mu_{P-I} > 0$. Table 3 shows these results. As it turns out, H_0 rejected at 90% significance. Hence, our model outperformed TEDPIX.

Table 3. Hypothesis test on excess mean return.

Mean portfolio return (%)	Mean TEDPIX return (%)	t-stat	$p-value$	result
0.8863	0.6404	3.203	0.06786	H_0 rejected

Figure 3 depicts the cumulative TEDPIX returns, and the out-of-sample portfolios returns result from the enhanced indexing model. As observed, the model portfolios returns outperformed the TEDPIX in the long term horizon. In the study period, the TEDPIX grew 6.5 times while our model portfolios could grow 13.65 times. This result confirms the outperformance of our model.

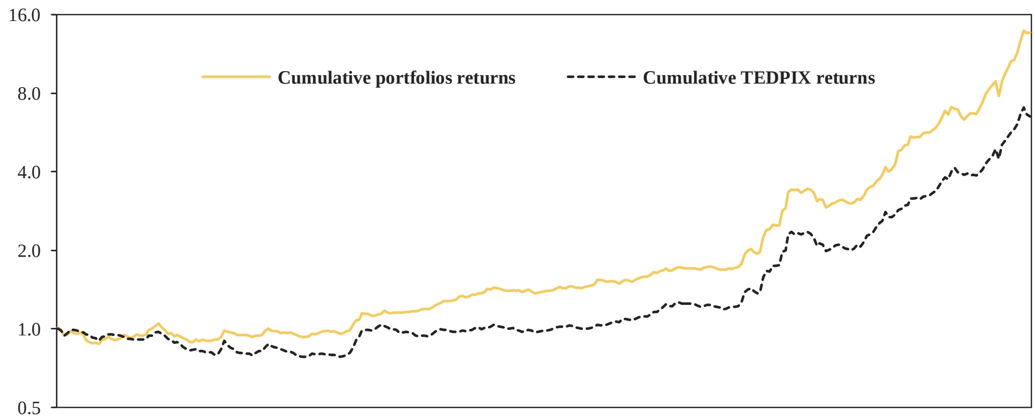

Figure 3. Cumulative returns of TEDPIX and the out-of-sample data portfolios were obtained from the research model for the twenty-six windows scheme.

5. Discussion and Conclusions

In recent years investors have focused on passive financial instruments more, and this causes massive growth in such passive investment. In 2017, the amount invested in passive funds (index funds) was more than $15 trillion around the world, (which is about 19% of the total asset under management); while it was about $3 trillion in 2003 that shows a vast growing interest in index funds (world asset management report, 2019). Thus, in this paper, we have studied investing in passive management strategies (index oriented) as an investment method with a return above index return.

The result shows that the research model was able to track the TEDPIX well, and through a 99% confidence, there was a most significant positive correlation. Also, the correlation of portfolio returns and TEDPIX return for all out-of-sample data was 0.8894. On the other hand, we have tested the portfolio excess mean return, and the results show that the model outperforms the TEDPIX. Besides, in a long-run horizon, the model returns have exceeded the TEDPIX returns.

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Notes

[1] If I_j is the jth realization of the index then $I_j(1+\alpha)$ is the jth realization of the index-plus-alpha portfolio.
[2] We apply the two-sided t_ρ test on null hypothesis $H_0: \rho = 0$ against the alternative hypothesis $H_a: \rho \neq 0$.