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Research Article

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A Bi-objective Capacitated Single-Allocation Hub Location Problem with Reliability Assumption on Paths

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Abstract

The hub location problems are highly crucial due to their applications in the transportation and distribution area. Today, the complexities of solving the real world problems using the single-objective techniques are challenging. For a more real model, the present study considers a bi-objective capacitated single-allocation hub location problem assuming the reliability of paths. In addition to the capacity, the fixed costs for the hubs are considered, as well. Furthermore, while minimizing the cost, the reliability of the weakest path is making maximized. Three mathematical models are proposed for this problem. The performance of single-objective models is evaluated and then, the proposed bi-objective model is solved using the -constraint method. In the present study, the fixed cost is calculated using two different methods: one is based on the distance from the center of mass and another one depends on the hub capacity. The results reveal that the third model with the fixed cost based on the distance from the center of mass has the best performance.

Keywords : Hub location problem; Capacitated hub; Reliability of path; Bi-objective optimization.

1 Introduction

The hubs are the special facilities being used as communication mediators among customers in order to economize the cost of transfer flow (or goods) in a network. In these problems, the hub centers are used instead of direct connections between the nodes. The flow is gathered from different origins and sent to various destinations after classification. The hubs can connect a large number of nodes together by a few communication lines. The hub location problems are widely used in the fields of transportation, telecommunication, emergency care and supply chain management [7].

Various types of hub location problems have been presented. These problems are different in their assumptions including the methods of the hubs selection, the customer allocation, the capacity constraint for the hubs, the cost calculation and their objects. The main hub location problems include p-hub median, p-hub center, the hub covering and hub location with fixed cost. The p-hub median location problem (pHMLP)

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aims to determine the location of p hubs and allocate customers to them in order to minimize the total cost of the flow transfer in the network. A p-hub center location problem (pHCLP) aims to minimize the maximum total cost of the flow transfer in a network. A hub covering problem intends to locate the hubs and allocate customers to them in order to minimize the total cost of establishing the hubs by allocating each customer to at least one hub (hub set covering location problem, HSCLP), or to transfer the highest possible flow in a network by assuming the establishment of a specified number of hubs (hub maximal covering location problem, HMCLP). The objective of a hub location problem with the fixed cost is to minimize the total cost including the costs of establishing the hub centers and transportation.

The hub location problems based on the structure of inter hub network are classified into complete and incomplete hub networks. A hub network is called complete if there is a direct connection between each two hubs and otherwise, it is called incomplete. In addition, the hub location problems, based on allocating the non-hub nodes to the hub ones, are categorized into single and multi-allocation ones. In single-allocation problems, each non-hub node is allocated to one hub, while the problem is called multi-allocation if a non-hub node can be connected with more than one hub center. In reality, the hub centers are limited in terms of the service level; therefore, a capacity constraint is considered for the hubs. The problems with some capacity constraints on hubs are called capacitated problems, versus the uncapacitated ones.

In the real world, the transportation systems can be disrupted by natural disasters or human mistakes. Such disruptions may weaken the system's performance (for instance, by presenting weak services and consequently increasing the costs). Therefore, designing a strong network is prioritized.

In the present study, a hub location problem (HLP) is considered with the fixed cost and facilities reliability. For the model being more real, the reliability of the transfer paths is maximized as much as possible. Therefore, in addition to minimizing the total cost, the reliability of the weakest path is maximized. Thus, in the present work, a bi-objective problem is studied which aims to minimize the total cost (including the costs of establishment and transportation) and maximize the reliability of the weakest path in a network. The reliability in this paper is based on its definition by Ebrahimi-Zadeh et al. [22]. The problem is considered to be capacitated, single-allocation, and have a complete hub network. The assumed model has many applications including the network programming for distribution of military equipments in a war, air and sea freights, and communication systems. These systems are so affected by the weather and regional conditions governing the inter-network connections.

The present paper includes a literature review of the previous studies on the hub location problems especially with the assumption of reliability (section 2), presentation of three mathematical models for the problem (section 3), analysis of the results of these models (section 4), and finally the conclusion.

2 Literature review

The various types of hub location problems have been introduced. Here, the literatures on the capacitated hub location problems (CHLPs) with the fixed cost are firstly reviewed, and then the disruption effects on the hub location problems are presented.

2.1 CHLPs with fixed cost

Most of the existing models are related to the uncapacitated HLPs. In these models, the capacity of hub centers is assumed to be unlimited in terms of amount of the input and output flows. O'Kelly [44] presented the first quadratic mathematical formulation for an uncapacitated HLP. In addition, he introduced the first HLP in which the fixed cost of establishment was considered in its objective function [45]. Campbell [11] suggested linearization for these models and formulated the other types of HLPs such as p-hub median, p-hub center and hub covering. Alumur and Kara [1]as well as Zanjirani-Farahani et al. [56] reviewed different types of HLPs.

In order to investigate the more real models, one should consider the limitations of hub centers for servicing. Campbell [11] presented the first mathematical model for a single-allocation capacitated hub location problem (SA-CHLP) with fixed cost. The location in the post centers is regarded as one of the applications of this problem. In each shift work, each servicer can categorize and arrange a limited number of parcel posts [24].

Aykin [3] examined a SA-CHLP with the probability of direct connections between the demand points. He also investigated the same problem with the assumption of establishing an assigned number of hub centers [4]. Ernst and Krishnamoorthy [24] presented a new formulation with smaller size for the SA-CHLP and found proper bounds for the problem by using two innovative methods and solved it by Branch Bound algorithm. Labbe et al. [34] used Branch Cut algorithm to solve the problem by presenting the valid inequalities. Costa et al. [19] evaluated two bi-objective SA-CHLPs; they aimed to minimize the total cost of transferring and establishing the hubs. The total and maximum servicing times in the hubs were minimized in the first and second problems, respectively. Contreras et al. [15] used the Lagrangean relaxation to find the appropriate upper and lower bounds of SA-CHLP and then, solved it. This method could solve the problems based on the samples of 200 points with appropriate accuracy. Correia et al. [17] assessed a development of the SA-CHLP with different capacity levels for the hubs. They offered different mixed-integer programming models for the problem and compared them in terms of size, performance of relaxed problems, valid inequalities and special preprocessing procedures. Correia et al. [17] demonstrated that some classic models of SA-CHLP are incomplete and do not provide appropriate answers for some examples. They modified these models by adding some constraints. Kratica et al. [33] presented a modified mixed-integer programming model for the SA-CHLP and used evolutionary methods for solving it. The presented model had a smaller size than the previous ones. Camargo and Miranda [9] examined the SA-CHLP assuming the existence of swarm in a network. In this nonlinear optimization model, in addition to minimizing the cost of establishment and transfer, the adverse effects of swarm in the hub centers are minimized, as well.

In addition to the above-mentioned investigations, some studies focused on the solving methods. Rodriguez and Salazar [49] used two Branch Cut algorithms for solving the SA-CHLP; one is based on Benders decomposition and another relies on LP relaxation. They assumed that the hub centers might not be fully connected to each other and the capacity and cost were considered for establishing both the hubs and the arcs. Chen [14] offered a simulated annealing algorithm with three levels for solving the SA-CHLP. This algorithm includes three phases for determining the number and location of the hubs and deciding on how to allocate the non-hubs to the hubs. Randall [48] used the ant colony algorithm to solve the SA-CHLP. Lin and Lee [35] applied the Lagrangian relaxation and game theory to solve the SA-CHLP. Camargo et al. [10] used a combination of the outer approximation and benders decomposition to solve this problem. Contreras et al. [16] applied a combination of the Lagrangian relaxation and Branch Price algorithm to solve a large-size SA-CHLP.

Campbell [11] presented the first mathematical formulation for a multi-allocation capacitated hub location problem (MA-CHLP) with fixed cost. Ebery et al. [21] offered a linear mathematical formula for the MA-CHLP and solved it using Branch Bound algorithm. Furthermore, they utilized an innovative algorithm to improve the upper bounds. Sasaki and Fukushima [53] modeled the MA-CHLP assuming the existence of a single hub in the flow transfer path from the origin to the destination and solved it using Branch Bound algorithm. They applied the Lagrangian relaxation to find the appropriate bound. Boland et al. [8] investigated the MA-CHLP properties and presented a simplified linear problem equal to the primary one without increasing the variables by new limitations. This linear problem improved the CPU time increasingly. Marin [38] focused on a linear integer programming formula for the MA-CHLP. He assumed that the flow between all the origin-destination pairs could be divided in different paths from origin to destination. In addition, the triangle inequality may not be established in calculating the length (or cost) of arcs in a network. Gelareh and Pisinger [26] evaluated the use of MA-CHLP in designing a sea transportation fleet. They offered 4 and 5-indices mixed-integer linear programming models for this problem and solved them using Benders decomposition.

2.2 HLPs with assumption of reliability

Assuming the increase of reliability in the location problems can improve services and enhance customer satisfaction. In addition, disruption in these problems causes undesirable performance and irreversible costs for the decision makers. Kim and O'Kelly [32] used the concept of reliability in HLPs for the first time. The objective function in their study was to maximize the total transferred flow in the network under the safety conditions. Zhalechian et al. [57] divided the risk into the failure and operational ones. The network disruptions in the failure and operational risks are related to the creation of a problem for a service center and the uncertainty of some parameters such as demands, cost of establishment and transportation, respectively.

Parvaresh et al. [46] examined a bi-level pHMLP assuming a disruption in the hubs performance. The transportation cost and the maximum of demand covering cost of the disrupted centers (in the worst state) were minimized in the first and second levels, respectively. They assumed that the maximum r hub centers were disrupted.

Geramianfar et al. [27] evaluated a bi-objective HLP aiming to decrease the swarm in a network. In their study, the first and second objective functions were used to minimize the total cost of establishment and transfer, and the total waiting time in hub centers, respectively. They considered a hub covering radius for allocating customers to the hub centers.

Bashiri and Rezanezhad [7] investigated a multi-objective hub covering location problem in which, besides the minimized transportation cost and maximum passage time in the network, the radius for hubs and allocating the demand points to closer hubs were limited.

Sadeghi and Tavakkoli-Moghaddam [52] assessed a reliable capacitated hub covering location problem. In order to enhance the network reliability, they assigned a flow transfer capacity to each link and considered a chance-constraint method to achieve the reliability constraint. Furthermore, they considered a covering radius for allocating the demand points to the hub centers.

Cardoso et al. [12] examined two types of uncapacitated HLP under the conditions of demand uncertainty and probability of hub centers failure. They used backup hubs and paths to decide on failure and disruption of the hubs.

Mohammadi et al. [41] designed a reliable logistic network by considering the probability of disruption in the hubs. They focused on the general disruption (hubs getting entirely out of reach) and the partial one (disruption in hub performance). Their model enhanced significantly the network reliability by a slight increase in the total cost.

Mohammadi et al. [42] evaluated a bi-objective fuzzy HLP by considering the waiting time in hub centers for receiving the service. They assumed that the hub centers might be disrupted. The first and second objective functions were used to minimize the total transportation cost and the maximum time of flow transfer between all the origin-destination pairs (by considering the probability of disruption in hub centers and necessity of customer waiting), respectively.

Azizi et al. [6] investigated an uncapacitated HLP at risk of failure. This model could determine the probability of failure and cost of rerouting in case of failure. They assumed that once a hub node stops normal operations, the entire demand initially served by this hub is allocated to a backup center.

Tran et al. [55] presented a mixed-integer nonlinear programming model for the uncapacitated HLP with the probability of hubs failure. The objective function aimed to minimize the total transportation cost and the fines caused by hub centers failure. A Tabu search algorithm was used to solve the sample problems.

Azizi [5] proposed an uncapacitated hub location model where a hub center was considered as a backup center for each demand point. The proposed model was of mixed-integer quadratic programming type which could solve the small and medium size samples in an acceptable time. The particle swarm optimization was used for the large-size samples. In another study, Rostami et al. [50] designed a bi-level formulation to achieve a reliable hub network by which that function is allocated to the backup hubs in the case of disrupted performance of a hub.

Chaharsooghi et al. [13] examined an uncapacitated multiallocation HLP assuming the probability of hub centers failure. They aimed to minimize the cost of establishing hubs, the mean cost of flow transfer, and the fines for the service disruption caused by the hub failure. They assumed that all the customers and flows of that hub are allocated to another hub in the case of a hub getting out of reach, and the allocation is not performed and the service disruption fine is added to the total cost if this allocation has a high cost. They showed that when the uncertainty in the operational status of hubs is considered, the number of hubs in optimal solution is greater than the classical counterpart in which it is assumed that the hubs are not subject to failure.

Ghodratnama et al. [30] proposed a bi-objective HLP by considering the swarm. The first and second objective functions aimed to minimize the total cost (including the hub establishment and transportation) and the total waiting time for processing the goods in factories and storehouses, respectively.

In addition to hub centers, disruption may be occurred in inter-hub links, as well. Kim and O'Kelly [32] attempted to consider the reliability of flow transfer paths in a network for the first time. Eghbali et al. [23] studied a reliable hub location problem. They examined a bi-objective problem in which the first and second objects were to minimize the total cost (including the hub establishment and transportation) and the total inter-network connections, respectively. Furthermore, they considered a constraint as the minimum reliability of paths between all the origin-destination pairs.

Ebrahimi-Zadeh et al. [22] proposed a bi-objective nonlinear programming for the single and multi-allocation HMCLPs. These models aimed to locate the hubs in such a way that both the coverage in network and the reliability of the weakest path were maximized. They attempted to reduce the complexity of the existing model by decreasing the numbers of constraints and variables. Then, they proposed a linear model and finally, solved it using the modified NSGA-II method. They investigated the effects of the second objective and showed that an increase in the importance of second objective results in selection of more reliable paths although the total covering demand may decrease. Pasandideh et al. [47] evaluated a bi-objective HMCLP. In their study, both the flow in the network and the total reliable flow were maximized. Furthermore, a probability constraint was considered for the cost of flow transfer between all the origin-destination pairs.

Ghaffari-Nasab [29] studied a CHLP assuming the reliability. In this study, a constraint was considered for the total cost (such as the flow transfer and hub establishment) in order to maximize the amount of flow with the assumption of reliability.

Nasiri et al. [43] examined a bi-objective pH-CLP assuming the reliability. Their first object was to minimize the total cost including the hub establishment (in demand points or candidate hub centers) and the flow transfer between the hub and non-hub nodes and the flow transfer in the network paths. The second object was to maximize the reliability of flow transfer paths.

Madani et al. [37] offered a reliable single-allocation HMCLP. The first object of their model was to maximize the expected value of cover flow and its second one was to reduce the swarm. In order to reduce the swarm in the network, the total difference of average expected flows passing through each hub and the average expected flows passing through all the hubs was minimized.

Sadat-Torkestani et al. [51] designed a reliable hub location model for the hierarchical network of a multi-modal transportation. They assumed that failure in the hub centers or the arcs linked to them occurs dynamically.

Fazel-Zarandi et al. [25] used a fuzzy goal programming to generalize the model of Kim and O'Kelly [32] . The proposed problem was a bi-objective one and the first and second objective functions were used to maximize the weighted reliability and also the minimum reliability in the network, respectively.

Concerning the HLPs with operational risk, three approaches (fuzzy optimization, stochastic and robust programming) were presented. Zhalechian et al.[57] reviewed them.

3 Mathematical modeling

In this section, three mathematical models are proposed. In order to propose the models, ideas were extracted from [22], [11] and [24] . The first model is a quadratic one and we show that the GAMS software could not solve it for large-size samples (subsection 4.3.1). The second model is presented to linearize the first one and finally, the third model is introduced to decrease the size of the second one. We show in section 4 that the third model is the best.

Introducing indices, parameters and decision variables

The indices, parameters and variables are defined as follows (all are common for three models):
Indices:
i and j are indices of origin and destination nodes, respectively, and m and k are indices for hub centers.

Parameters

$N = \{1, 2, ..., n\}$: the set of demand points,
d_{ij}: indicates the distance between origin i to destination j (it is assumed that d_{ij} is verified in triangle inequality and for each i, $d_{ii} = 0$),
W_{ij}: flow amount from origin i to destination j,
C_{ij}: transportation cost per flow from node i to node j,
C_{km}: cost of transportation between two hubs k and m,
O_i: total export flow from node i ($O_i = \sum_j W_{ij}$),
F_k: fixed cost of establishing hub in candidate location k,
P_{ij}: security of arc i to j,
α: discount factor,
M: a big number.

Decision variables

$X_{ik} \in \{0, 1\}$: binary variable, which is equal to 1 if node i is allocated to a hub located at node k and equal to 0, otherwise.
$X_{kk} \in \{0, 1\}$: binary variable, which is equal to 1 if node k is hub and equal to 0,otherwise.
$S \geq 0$: security of the weakest path.
It is assumed that demand and distance between two nodes are given and they are deterministic parameters. The hubs are selected among these demand nodes. The path between each pair of origin node i to destination node j is as $i \to k \to m \to j$, in which k and m are the nodes of hub allocating to i and j, respectively. The cost of transportation for each unit of flow from origin i to destination j is the total cost of flow path. $0 \leq \alpha \leq 1$ indicates the discount factor. This parameter is one of the most effective parameters in HLPs. O'Kelly [44] used discount factor α for inter-hub paths for the first time. He multiplied the cost of inter-hub paths to parameter α to show the effect of inter-hub flow transportation. Through utilizing this parameter, customers are encouraged to use the hub center.

The first mathematical model for problem

The first model's idea is according to the study of Ebrahimi-zadeh et al. [22]. Considering the presented assumptions, the mathematical model is as follows:

$$P_1: \quad Min \sum_i \sum_j \sum_k \sum_m W_{ij}(C_{ik} + \alpha C_{km} + C_{mj})X_{ik}X_{mj} + \sum_k F_k X_{kk} \tag{3.1}$$

$$Max \;\; S \tag{3.2}$$

$$s.t \quad S \leq (X_{ik}X_{mj})(P_{ik}P_{km}P_{mj}) + M(1 - X_{ik}X_{mj}) \forall i,j,k,m \tag{3.3}$$

$$X_{ik} \leq X_{kk} \qquad \forall i,k \tag{3.4}$$

$$\sum_k X_{ik} = 1 \qquad \forall i \tag{3.5}$$

$$\sum_k \sum_m X_{ik}X_{mj} = 1 \quad \forall i,j \tag{3.6}$$

$$\sum_i O_i X_{ik} \leq b_k X_{kk} \qquad \forall k \tag{3.7}$$

$$X_{ik} \in \{0,1\} \qquad \forall i,k \tag{3.8}$$

$$S \geq 0 \tag{3.9}$$

The first objective function aims to minimize the total cost which is including two parts. The first part is related to the cost of transportation between each pair of origin node i and destination node j (via hub nodes located at nodes k and m), and the second part is related to the cost of establishing hub. The second objective function attempts to maximize the reliability of the weakest path in the network. Moreover constraint (3.3) determines an upper bound for the reliability of the weakest path.

Reliability of the path from origin node i to destination node j is defined as multiplication of reliability of arcs in path $i \to k \to m \to j$. If the connection of this path is established, an upper bound for S is provided. However, this constraint converts to an excessive constraint when there is not any connection in this path. S is a lower bound for the reliability of paths and by considering the second objective function, S becomes the safety of the most insecure path. Constraint (3.4) means that node i could be allocated to node k if node k is a hub. Constraint (3.5) guarantees that each non-hub node only could be allocated to one hub node. This constraint indicates that the problem is a single-allocation. Constraint (3.6) declared that there are hub nodes at k and m for two nodes i and j, which flow is transferred from node i to node j via them. Constraint (3.7) is the capacity limitation of hubs. The flow amount which entered to hub node k should be less than or equal to hub capacity. Constraints (3.8) and (3.9) indicate the domain of variables X_{ik} and S.

Constraint (3.10) is a linear equivalent to non-linear constraint (3.3) [22]. Therefore, the modified model is obtained by replacing constraint (3.3) with (3.10).

$$S \leq \left(\frac{X_{ik} + X_{mj}}{2}\right)(P_{ik}P_{km}P_{mj}) + M\left(1 - \left(\frac{X_{ik} + X_{mj}}{2}\right)\right) \qquad \forall i,j,k,m \tag{3.10}$$

Model P_1 is a mixed-integer nonlinear programming type and also it has $n^2 + n$ variables and $n^4 + 2n^2 + 2n$ constraints. The first objective function and constraint (3.6) of this model are quadratic. To linearize P_1, the second model (P_2) is presented.

The second mathematical model for the problem

The idea of the second model is extracted from [11]. The parameters and variables are considered the same as previous part. In addition, variable X_{ijkm} is defined as follows:

$X_{ijkm} \in \{0,1\}$: it is a binary variable, which is equal to 1 if the flow from origin node i to destination node j transfer from hub nodes at k and m, and it is equal to 0, otherwise.

Given the noted assumptions, the second model could be proposed as follows:

$$P_2:$$
$$Min \sum_i \sum_j \sum_k \sum_m W_{ij}(C_{ik} + \alpha C_{km} + C_{mj})X_{ijkm} + \sum_k F_k X_{kk} \quad (3.11)$$
$$Max \;\; S$$
$$s.t \quad (3.4), (3.5), (3.7) - (3.10)$$
$$\sum_k \sum_m X_{ijkm} = 1 \quad \forall i, j \quad (3.12)$$
$$\sum_k X_{ijkm} = X_{jm} \quad \forall i, j, m \quad (3.13)$$
$$\sum_m X_{ijkm} = X_{ik} \quad \forall i, j, k \quad (3.14)$$
$$X_{ijkm} \in \{0, 1\} \quad \forall i, j, k, m \quad (3.15)$$

The objective function (3.11) includes two parts as the first model. The first and second parts minimize the cost of transporting from node i to node j (in a way that flow transfers from hub nodes at k and m) and establishing hubs, respectively. Constraint (3.12) indicates that only when flow could transfer from node i to node j, which it passes from hub nodes at k and m. Constraint (3.13) declares that if node j is allocated to hub node m then there is one node k which i is allocated to it. Constraint (3.14) presents an interpretation like (3.13). Constraint (3.15) indicates the definition domain of decision variable X_ijkm. The objective function (3.2) and other constraints are defined as previous.

The second model is mixed-integer linear programming and has $n^4 + n^2 + n$ variables and $n^4 + 2n^3 + 2n^2 + 2n$ constraints. The third model P_3 is introduced to decrease the size of model P_2.

The third mathematical model for the problem

The idea of the third model is taken from [24]. Variable X_ijkm is eliminated from the second model and replaced with three indices variable Y^i_{km} which is defined as follows:

Y^i_{km}: it refers to the flow amount originated from node i, in which it passes from hub nodes k and m in order to reach the destination.

With regard to the problem assumptions and defined variables, the third mathematical model is as follows:

$$P_3:$$
$$Min \sum_i \sum_k C_{ik}(O_i + D_i)X_{ik} + \sum_i \sum_k \sum_m \alpha C_{km} Y^i_{km} + \sum_k F_k X_{kk} \quad (3.16)$$
$$Max \;\; S$$
$$s.t \quad (3.4), (3.5), (3.7) - (3.10)$$
$$\sum_m Y^i_{km} - \sum_m Y^i_{mk} = O_i X_{ik} - \sum_j W_{ij} X_{jk} \quad \forall i, k \quad (3.17)$$
$$Y^i_{km} \geq 0 \quad (3.18)$$

The objective function (3.16) of this model includes three parts: the cost of flow transfer between hub and non-hub nodes, the cost between hubs and the fixed constant of establishing hubs. The objective function (3.2) and constraints (3.4), (3.5) and (3.7)-(3.10) are defined like before. Constraint (3.17) is flow balance equation, which indicates the balance between supply and demand at each node i if it is allocated to hub node k. Constraint (3.18) displays the domain of Y^i_{km}. Correia et al. [18] demonstrated that this model does not work for some samples accurately. Particularly, the amount of Y^i_{kk} in formula which is introduced by Ernst and Krishnamoorthy [24] is always 0. Therefore, to modify the model, the following constraint is added to this model:

$$\sum_{m:\; m \neq k} Y^i_{km} \leq O_i X_{ik} \quad \forall i, k \quad (3.19)$$

Constraint (3.19) guarantees that $Y^i_{km} = 0$ if node i is not allocated to the hub node at k. This constraint avoids creating the answers and paths, which do not exist in the hub network. Thus, the third model is completed by adding constraint (3.19). P_3 is linear programming and has $n^3 + n^2 + n$ variables and $n^4 + 3n^2 + 2n$ constraints. The size of the third model is smaller than that of the second model.

4 Solving the models and analyzing the results

Multi-objective optimization (MOO) is a simultaneous process of two or more objective functions (usually opposite) with some constraints. For instance:

$$\begin{aligned} Min\ \ F(x) &= \{f_1(x), f_2(x), ..., f_N(x)\} \\ s.t\ \ g(x) &\le 0 \\ h(x) &= 0 \\ x &\in \mathbb{R} \end{aligned}$$

MOO could not achieve an answer simultaneously while all objective functions are optimized. In such problems, despite single-objective problems, a set of answers is obtained, which is called Pareto solutions. Pareto solutions in multi-objective are a set of non-dominated points which dominate all the other solutions. Assume that x_1 and x_2 are two solutions for a multi-objective problem. Solution x_1 dominates x_2 if x_1 is not worse than x_2 in none of the functions and x_1 is strongly better than x_2 at least in one of the objects [20]. It means:

$$f_i(x_1) \le f_i(x_2) \qquad \forall i = 1, 2, ..., N \tag{4.20}$$

$$f_j(x_1) < f_j(x_2) \qquad \exists j \in \{1, 2, ..., N\} \tag{4.21}$$

CHLP is a special case of the proposed bi-objective problem and because CHLP is NP-hard, the discussed bi-objective problem is also NP-hard. Therefore, achieving reliable solutions may not be easy by using optimization software. In the present study, ε-constraint method is used to solve the problem. This method was introduced by Haimes et al. [31] for the first time. This method is probably one of the well-known methods for achieving Pareto solutions of small-size multi-objective discrete optimization problems [40]. Tavakoli-Moghadam et al. [54] , Bashiri and Rezanezhad [7] , and Ghezavati and Hossseinifar [28] used ε-constraint method to solve their multi-objective facility location problems.

4.1 ε-constraint method

ε-constraint method is one of the known methods for solving multi-objective problems. In this method, one of the objective functions is considered as the main objective and the other objective are considered as the constraints. The definition of right values (e_i) of constraints is an important task. In order to obtain the right values of these constraints, Payoff table is created. After obtaining values e_i, the single-objective problem is solved and Pareto solutions are obtained.
Consider the following bi-objective problem. The bi-objective problem could be converted to the single-objective problem as follows. Here, objective function f_1 is considered as the main objective function [39]:

$$\begin{aligned} &Max\ \ f_1(x) & & Max\ \ f_1(x) \\ &Max\ \ f_2(x) & \Rightarrow \quad & s.t\ \ x \in \Omega \\ &s.t\ \ x \in \Omega & & \quad f_2(x) \ge e_2 \end{aligned}$$

In minimizing the problem, the sign " $\ge$ " is converted to " $\le$ ". Payoff table is needed to obtain the values of $e_i's$. This table includes four components for the bi-objective problem. Their components are as follows:

$$Payoff\ \ Table = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

a_{11} and a_{22} are obtained from single-objective function P_4 and P_5, respectively:

$$P_4)\ \ a_{11} = Max\ \ f_1(x) \quad s.t\ \ x \in \Omega \qquad\qquad P_5)\ \ a_{22} = Max\ \ f_2(x) \quad s.t\ \ x \in \Omega$$

a_{12} and a_{21} are obtained from single-objective function P_6 and P_7, respectively:

$$P_6)\ \ a_{12} = Max\ \ f_1(x) \quad s.t\ \ x \in \Omega,\ \ f_2(x) = a_{22} \qquad\qquad P_7)\ \ a_{21} = Max\ \ f_2(x) \quad s.t\ \ x \in \Omega,\ \ f_1(x) = a_{11}$$

In order to achieve the different values of e_2, the distance between a_{12} and a_{22} is divided into equal parts. To obtain Pareto solutions set for the bi-objective problem, the single-objective problem should be solved with one of $e_i's$ each time. In ε-constraint method, the number of these distances is equal to ε. Due to the highly effect of ε value on Pareto solutions, this method is known as ε-constraint.

4.2 Providing the data

In the present study, data set of CAB was used for investigating the function of models and comparing their computational results. These data belong to American Airline [44]. The necessary data should be provided due to unavailability of some parameters. Ebery et al. [21] presented some formulations for calculating the capacity and fixed cost of CHLP which were used here to provide the required parameters. For a problem with n demand points and p hub nodes, the capacity of node i is calculated as follows:

$$b_i = (\frac{n}{p} + \frac{3d_i o_i}{5d_m o_m})O_i \tag{4.22}$$

Where d_i, O_i, d_m and O_m are the distance of node i from the center of mass, export flow from node i, maximum value of d_i and the maximum value of O_i, respectively. p is an integer, which controls the number of hubs. In addition, the following relationships are used for calculating the fixed cost of establishing hubs.
The fixed cost of establishing hubs based on the capacity of each node:

$$F_i^c = f_0(5(\frac{b_i + o_i}{b_m + o_m}) + \frac{1}{2}) \tag{4.23}$$

The fixed cost of establishing hubs based on distance from the center of mass:

$$F_i^d = f_0(1 - \frac{3d_i}{d_m}) \tag{4.24}$$

in which b_m is the maximum capacity of nodes and f_0 is defined as follows:

$$f_0 = (\sum_i \sum_j (C_{ih} + C_{hj})W_{ij} - \sum_i \sum_j \alpha C_{ij} W_{ij})/p \tag{4.25}$$

Here, f_0 represents the scaled difference in objective value between two following scenarios:

1. Central node h is a hub.
2. All nodes are hub.

h is the closest node to the center of mass. In the first part of the equation (4.25), collecting and distributing the flow are performed only by one hub node and in the second part, all the flows are transferred from origin to destination directly.
It should be noted that geographical coordinates of the center of mass (X_h, Y_h) are calculated as follows [36]:

$$X_h = \frac{\sum_{j=1}^n o_j X_j}{\sum_{j=1}^n o_j} \tag{4.26}$$

$$Y_h = \frac{\sum_{j=1}^n o_j Y_j}{\sum_{j=1}^n o_j} \tag{4.27}$$

In the above equations,(X_j, Y_j) are the geographical coordinates of j, which are taken from *www.latlong.net*. After finding the coordinates of the center of mass, the distance of each node from it is calculated and the closest node is considered as the center of mass. For CAB data, the closest node to the center of mass is node 21.
On the other hand, CAB dataset does not include the data related to the reliability of the paths. Here the elements of safety matrix are provided between $[0.7, 1]$ randomly. In addition, the probability of safe arrival of cargo from one node to the same node is imminent. Therefore, the elements of the main diagonal of this matrix are always equal to 1. In the present study, the matrix of reliability is assumed to be symmetric.

4.3 Computational results

The three presented models are analyzed in two steps:

1. Firstly, the models are investigated without considering the reliability. It means that the second objective function and the constraint for determining upper bound are eliminated. Therefore, there are three single-objective models. This elimination is performed to analyze the models, declare the importance of capacity parameters and determine the fixed cost of establishing hubs, as well as to determine an appropriate method for calculating the parameters.

2. After solving the single-objective models, the most appropriate one is selected and then, the bi-objective problem with assumed parameters is analyzed.

4.3.1 Results based on single-objective models

Since two different methods were proposed for calculating fixed cost, computational results of each one are presented separately. In this section, the effect of hub capacity, discount factor, number of hub centers, and fixed cost on the problem are analyzed.

The three single-objective models were coded by GAMS 24.1.2 in a system of Intel core i5 and RAM4G. Baron and CPLEX solvers were used for solving the first nonlinear model and the two other ones, respectively. *ncp* sign means a sample with n nodes, p hubs and fixed cost based on capacity (c sign). Similarly, *ndp* sign means a sample with n nodes, p hubs and fixed cost based on distance from the center of mass.

4.3.1.1. Analyzing single-objective models considering relation (4.23)

In this part, solving the three single-objective capacitated models is examined in which their fixed cost is calculated based on the capacity (relation (4.23)). The results are given in Table 1. As seen, the software could not solve nonlinear model of P_1 (the software output for this model was no solution found). In addition, the objective function value and allocating nodes and hub locations are identical for the second and third models and the time required for solving the third model is less than that of the second model. Therefore, the third model is better than the second one in terms of time. Figure 1 represents the objective function changes compared with those of discount factor and hub number. As shown in 1 and Figure 1, the cost is reduced by increasing the hub number. Unexpectedly, the minimum and maximum costs are related to the least and most discounts, i.e. $\alpha = 0.8$ and $\alpha = 0.2$, respectively. This is due to the domination of fixed cost of establishing hubs and transporting between the hub and non-hub nodes over the cost of transporting between the hub nodes. Figure 2 displays the determined locations of hubs and how the nodes were allocated for the sample of $25c7$ and $\alpha = 0.8$. The hub and non-hub nodes are shown as squares and circles,

Table 1: Results of solving three single-objective models (fixed cost was calculated based on relation (4.23))

α	sample	The best value for P_1	The best value foe P_2 and $P_3 (\times 10^{10})$	CPU for P_2 (in sec)	CPU for P_3 (in sec)	Hubs
0.2	$25c3$	—	2.873421	2616	17	8, 9, 25
	$25c5$	—	2.626912	3036	7	4, 22, 25
	$25c7$	—	2.452331	2199	18	10, 17, 21, 22
0.4	$25c3$	—	2.679741	3263	25	8, 9, 25
	$25c5$	—	2.514920	3304	25	4, 22, 25
	$25c7$	—	2.360280	2412	42	10, 17, 21, 22
0.6	$25c3$	—	2.486051	3063	21	8, 9, 25
	$25c5$	—	2.390541	6043	75	4, 8, 24, 25
	$25c7$	—	2.265650	1211	58	7, 17, 21, 22
0.8	$25c3$	—	2.292373	3379	43	8, 9, 25
	$25c5$	—	2.219304	5553	94	4, 9, 25
	$25c7$	—	2.160052	1699	40	4, 17

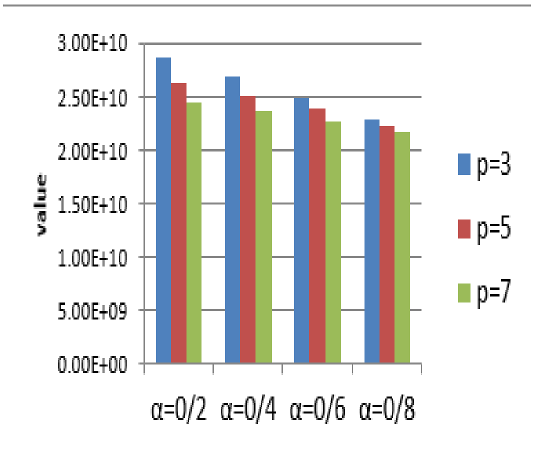


Figure 1: Comparing objective functions in terms of changes of hub number and discount factor (fixed cost was calculated based on relation (4.23)

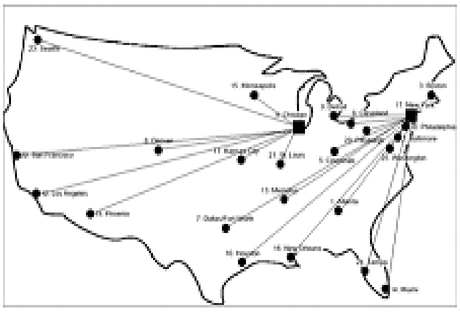


Figure 2: Hub location and allocations for sample $25c7$ and discount factor of 0.8 (fixed cost was calculated based on relation (4.23)

respectively. There is a connection between hubs 4 and 17 which is not shown for the allocations to be well observed. Additionally, in solving this model, some nodes close to hub 4 (for instance, node 9) were allocated to hub 17 due to the high capacity of hub 17. It means that the hub node may not respond to the demands of nodes close to itself since its capacity is limited.

4.3.1.2 Analyzing single-objective models considering relation (4.24)

Now, we examine the single-objective models of P_1, P_2 and P_3 in which fixed cost is calculated based on distance from center of mass (relation (4.24)). Like calculating fixed cost based on capacity, GAMS software could not calculate P_1 model. The results of P_2 and P_3 are given in Table 2. Their comparison demonstrates that objective function value and allocating nodes and hub locations are identical for the second and third models. Nodes 12 and 17 are hub in all the samples. Since the objective function values, hubs and allocation are identical for the two models and CPU time for solving the models are highly different, the third model is more appropriate for the considered problem.

Figure 3 displays the objective function changes compared with those of discount factor and selected hub number. As shown in Table 2 and Figure 3, cost is reduced by increasing the number of hubs. However, unlike before, the least and most optimized values are related to the most and least discounts, $\alpha = 0.2$ and $\alpha = 0.8$, respectively. However, the discount factor plays an important role in calculating fixed cost based on the distance from center of mass and fixed cost of establishing hubs and transporting between the hub and non-hub nodes does not dominate over cost of transporting between the hub nodes.

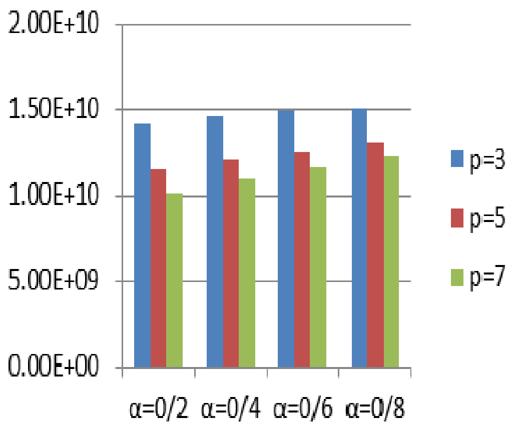


Figure 3: Comparing objective functions in terms of changes of hub number and discount factor (fixed cost was calculated based on relation (4.24)

Figure 4 shows the hub locations and allocations for sample $25d8$ and $\alpha = 0.2$. Again, inter-hub paths were not depicted for the images being clearer. As seen, the locations of hub centers and allocating nodes to them are more reasonable than those of state in which cost was calculated based on capacity.

Comparing the results in Tables 1 and 2 and ob-

Table 2: Results of calculating three single-objective models (fixed cost was calculated based on the distance from center of mass)

α	sample	The best value for P_1	The best value foe P_2 and $P_3 (\times 10^{10})$	CPU for P_2 (in sec)	CPU for P_3 (in sec)	Hubs
0.2	25d3	—	1.415380	1253	1	12, 17
	25d5	—	1.153642	1050	1	4, 12, 17
	25d7	—	1.011390	738	1	4, 12, 14, 17
0.4	25d3	—	1.454280	1141	2	12, 17
	25d5	—	1.204930	1140	1	4, 12, 17
	25d7	—	1.091310	954	2	4, 12, 17
0.6	25d3	—	1.493180	1570	8	12, 17
	25d5	—	1.256210	1702	1	4, 12, 17
	25d7	—	1.159870	1380	2	4, 12, 17
0.8	25d3	—	1.509750	1962	16	4, 12, 17
	25d5	—	1.307500	1631	2	4, 12, 17
	25d7	—	1.228430	1150	3	4, 12, 17

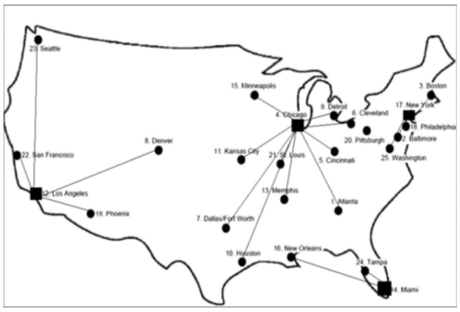


Figure 4: Hub location and allocating non-hub to hub nodes for sample 25d8 and $\alpha = 0.2$ (fixed cost was calculated based on relation (4.24))

jective function values reveals a decrease in the total cost as fixed cost is calculated based on the distance from the center of mass. In addition, the single-objective model P_3 has the best performance in terms of the running time. Therefore, the bi-objective model P_3 in which fixed cost is calculated based on the distance from center of mass is examined.

4.3.2 Numerical calculation based on bi-objective model

In this section, we investigate the results of solving the bi-objective model of P_3 in which fixed cost is calculated based on the distance from center of mass. Since the problem is bi-objective, Pareto solution set is obtained instead of one answer. In order to determine the most appropriate answer among the answer set, two weighting indexes, namely the weighted sum method (WSM) and analytic hierarchy process (AHP), are used. The WSM and AHP are among the most common methods for selecting the most appropriate Pareto answer [2]. The score is calculated for each Pareto solution by summation of the normalized values of each objective multiplied by its relative weight. Given +1 and −1 coefficients for minimizing and maximizing the objective functions, respectively, the least values for WSM_i and AHP_i suggest more priority of Pareto answer for being selected.

In the weighted sum method for Pareto answer i,

WSM_i is obtained via the following formula [20]:

$$WSM_i = \sum_{n=1}^{N} (+/-) w_n \left(\frac{z_n^i}{z_n^{max}}\right) \quad (4.28)$$

In the rating method based on the AHP for Pareto answer i, AHP_i is obtained via the following formula [20]:

$$AHP_i = \sum_{n=1}^{N} (+/-) w_n \left(\frac{z_n^i}{z_n^{total}}\right) \quad (4.29)$$

Where N and w_n are the number of existing objective functions and weight of n^{th} objective function, respectively. Z_n^i, Z_n^{max} and Z_n^{total} are n^{th} objective function value in Pareto answer i, the maximum and total of Z_n^i values, respectively. Values of Z_n^{max} and Z_n^{total} are calculated as follows:

$$Z_n^{max} = Max_i Z_n^i$$
$$Z_n^{total} = \sum_i Z_n^i$$

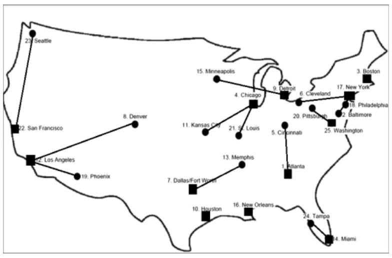
The total weight for the two methods should be equal to 1. In the present study, it is supposed that for decision makers, the safe arrival of cargo is more important than total cost. Therefore, weights of $\frac{1}{3}$ and $\frac{2}{3}$ are assumed for the first and second objects, respectively.

Tables (3)-(6) present the results of solving bi-objective model P_3, in which fixed cost was calculated based on the distance from center of mass, for different values of discount factor. The first, second, third and fourth columns (from left to right) display the sample name, the number of Pareto answers and the optimized value of the first and second functions, respectively. The fifth, sixth and seventh columns are related to the values of WSM_i and AHP_i, the time required for solving the model (in second) and the selected hubs, respectively. The best Pareto among existing Pareto set is the one indicated by an asterisk (*).

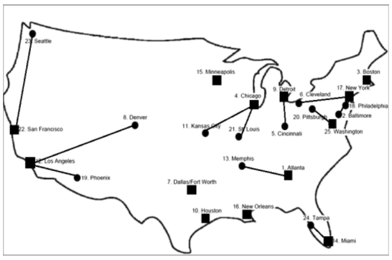
As seen in Table 3, the time required for solving the samples decreases and the number of selected hubs increases with possibility of increasing the hub centers. On the other hand, the reliability has a direct relationship with cost. Figure 5 displays allocating the non-hub nodes to

Table 3: Results of solving bi-objective model P_3 for $\alpha = 0.2$

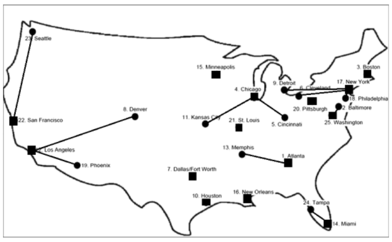
height3*sample	3*number of Pareto solutions	3*optimized value of the first function (10^9)	3*optimized value of the second function	indexes		3*CPU (sec)	3*Hubs
				2*$WSM_i(\times(-1))$	2*$AHP_i(\times(-1))$		
$25d16$	1	9.166412	0.750	0.2253	0.0436	105	1, 3, 4, 7, 9, 10, 12 , 14, 16, 17, 22 , 25
	2	9.168074	0.610	0.2628	0.0504		
	3	9.354853	0.630	0.2753	0.0528		
	4	9.386704	0.670	0.3118	0.0594		
	5	9.537325	0.700	0.3349*	0.0637*		
	6	9.860268	0.710	0.3333	0.0635		
$25d17$	1	9.250346	0.530	0.1830	0.0364	77	1,3,4,7,9,10 ,12,14,15,16 ,17,22,25
	2	9.295249	0.610	0.2566	0.0499		
	3	9.338525	0.670	0.3114	0.0599		
	4	9.493233	0.680	0.3155	0.0607		
	5	9.508446	0.690	0.3244	0.0623		
	6	9.798310	0.710	0.3333*	0.0641*		
$25d18$	1	10.18571	0.710	0.3333	0.0512	71	1,3,4,7,10,12 ,14,15,16,17,20 ,21,22,25
	2	9.382654	0.590	0.2469	0.0381		
	3	9.529038	0.630	0.2797	0.0431		
	4	9.560969	0.670	0.3162	0.0486		
	5	9.592871	0.680	0.3246	0.0499		
	6	9.647687	0.690	0.3322	0.0510		
	7	9.859953	0.700	0.3346*	0.0514*		



(a) $a)25d16$

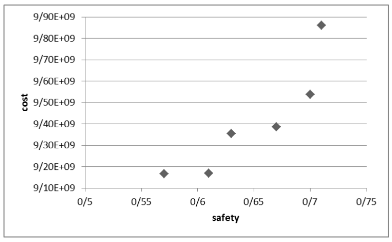


(b) $b)25d17$

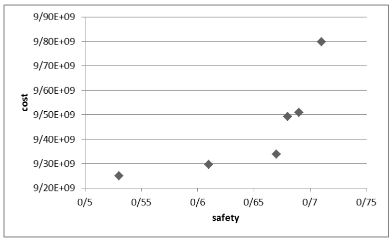


(c) $c)25d18$

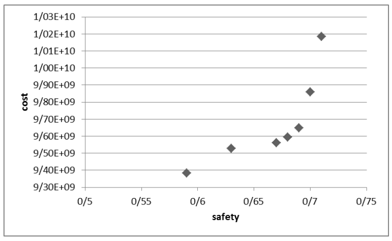
Figure 5: Hubs and allocating non-hub nodes to hub centers for bi-objective problem with $\alpha = 0.2$, $a)25d16$, $b)25d17$ and $c)25d18$



(a) $a)25d16$



(b) $b)25d17$



(c) $c)25d18$

Figure 6: Values of Pareto answers for bi-objective problem with $\alpha = 0.2$, $a)25d16$, $b)25d17$ and $c)25d18$

the hub nodes and the hubs location for samples of $25d16$, $25d17$ and $25d18$, with $\alpha = 0.2$. Figure 6 shows Pareto borders for the samples with $\alpha = 0.2$. As seen, the total cost increases by increasing the reliability. Table 4 presents Pareto answers of bi-objective model P_3, in which the total cost was calculated based on the distance from center of mass, with $\alpha = 0.4$. Figure 7 displays allocating the non-hub nodes to the hub ones and the hubs location for the bi-objective problem of $25d16$ with $\alpha = 0.4$. Allocating the non-hub nodes to the hub ones and the hubs location for the problems of $25d17$ and $25d18$ with the discount factor of 0.4 and 0.2 are similar, as shown in Figure 5. Pareto answers of bi-objective problem P_3 model with $\alpha = 0.6$ are given in Table 5. Allocating the non-hub nodes to the hub ones and the hubs location for the bi-objective

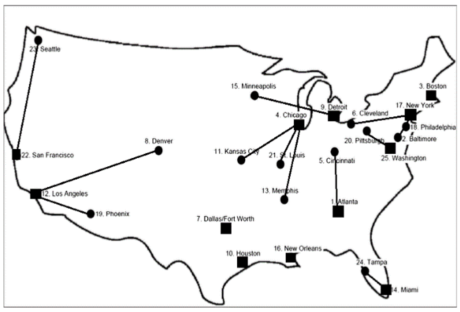


Figure 7: Hubs and allocating non-hub nodes for bi-objective problem, sample of $25d16$ with $\alpha = 0.4$

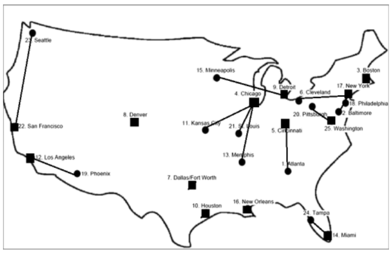
Table 5: The results of solving bi-objective model of P_3 with $\alpha = 0.6$

height3*sample	3*number of Pareto solutions	3*optimized value of the first function (10^{10})	3*optimized value of the second function	indexes		3*CPU (sec)	3*Hubs
				2*$WSM_i(\times(-1))$	2*$AHP_i(\times(-1))$		
25d16	1	1.088533	0.610	0.2528	0.0923	158	1, 3, 4, 7, 8, 9, 10
	2	1.106871	0.700	0.3319	0.1202		, 12, 14, 16, 17
	3	1.133981	0.710	0.3333*	0.1208*		, 22, 25
25d17	1	1.104362	0.530	0.1690	0.0532	100	1,3,4,7,8,9,10
	2	1.110675	0.670	0.2986	0.0886		,12,14,15,16
	3	1.115566	0.690	0.3159	0.0934		,17,22,25
	4	1.120173	0.710	0.3333*	0.0982*		
25d18	1	1.111183	0.590	0.2322	0.0651	74	1,3,4,7,8,9,10
	2	1.118737	0.670	0.3052	0.0846		11,12,14,15,16
	3	1.126123	0.700	0.3312	0.0915		,17,20,22,25
	4	1.151142	0.710	0.3333*	0.0921*		

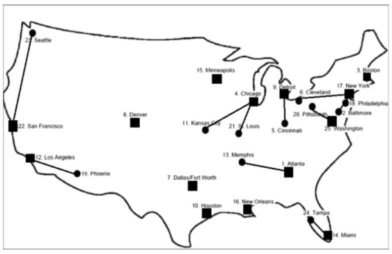
Table 4: The results of solving bi-objective model P_3 for $\alpha = 0.4$

height3*sample	3*number of Pareto solutions	3*optimized value of the first function (10^{10})	3*optimized value of the second function	indexes		3*CPU (sec)	3*Hubs
				2*$WSM_i(\times(-1))$	2*$AHP_i(\times(-1))$		
25d16	1	1.003921	0.610	0.2574	0.0575	96	1, 3, 4, 7, 9, 10, 12
	2	1.023671	0.630	0.2700	0.0602		, 14, 16, 17, 22
	3	1.027301	0.670	0.3064	0.0680		, 25
	4	1.031581	0.700	0.3333	0.0738		
	5	1.061221	0.710	0.3333*	0.0739*		
25d17	1	1.021502	0.530	0.1740	0.0360	97	1,3,4,7,9,10
	2	1.026573	0.610	0.2475	0.0494		,12,14,15,16
	3	1.027472	0.670	0.3036	0.0596		,17,22,25
	4	1.038774	0.680	0.3094	0.0607		
	5	1.039392	0.690	0.3186	0.0624		
	6	1.052136	0.710	0.3333*	0.0652*		
25d18	1	1.031362	0.590	0.2381	0.0336	72	1,3,4,7,10,12
	2	1.044043	0.620	0.2624	0.0369		,14,15,16,17,20
	3	1.044262	0.630	0.2717	0.0382		,21,22,25
	4	1.045100	0.670	0.3090	0.0432		
	5	1.046732	0.670	0.3085	0.0431		
	6	1.054721	0.690	0.3248	0.0453		
	7	1.060526	0.700	0.3325	0.0464		
	8	1.088301	0.710	0.3333*	0.0465*		

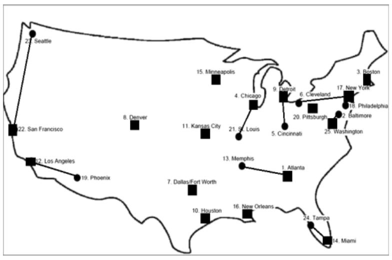
problems of $25d16$, $25d17$ and$25d18$ with $\alpha = 0.6$ are presented in Figure 8.



(a) $a)25d16$



(b) $b)25d17$



(c) $c)25d18$

Figure 8: Hubs and allocating non-hub nodes for bi-objective problem with $\alpha = 0.6$

Table 6 presents Pareto answers of P_3 bi-objective problem with the discount factor of 0.8. Allocating the non-hub nodes to the hub ones and the hubs location for the samples of $25d16$, $25d17$ and $25d18$ with$\alpha = 0.8$are similar to those of $25d16$ with $\alpha = 0.4$,$25d17$ with $\alpha = 0.4$and$25d18$ with $\alpha = 0.6$, respectively.

Table 6: Results of solving bi-objective model of P_3 with $\alpha = 0.8$

height3*sample	3*number of Pareto solutions	3*optimized value of the first function (10^{10})	3*optimized value of the second function	indexes		3*CPU (sec)	3*Hubs
				2*$WSM_i(\times(-1))$	2*$AHP_i(\times(-1))$		
$25d16$	1	1.164622	0.530	0.1738	0.0560	125	1, 3, 4, 7, 8, 9, 10 , 12, 14, 16, 17 , 22, 25
	2	1.165051	0.610	0.2488	0.0769		
	3	1.174064	0.700	0.3308*	0.0998*		
	4	1.198851	0.710	0.3333*	0.1006*		
$25d17$	1	1.175103	0.700	0.3262	0.1105	93	1,3,4,7,8,9,10 ,12,14,15,16 ,17,22,25
	2	1.182719	0.700	0.3241	0.1098		
	3	1.183146	0.710	0.3333*	0.1130*		
$25d18$	1	1.182664	0.590	0.2268	0.0864	76	1,3,4,7,8,9,10 11,12,14,15,16 ,17,20,22,25
	2	1.186625	0.700	0.3290	0.1227		
	3	1.204827	0.710	0.3333*	0.1243*		

Analyzing the results of solving the bi-objective model of P_3

Comparison of the results presented in Tables (3)-(6) brings about the following results:

1- The minimum cost is related to the maximum discount of $\alpha = 0.2$. Therefore, the cost increases by increasing the discount. In addition, a decrease occurs in the numbers of Pareto solutions.

2- In all cases, the best Pareto solutions determined by WSM and AHP indexes are identical. That Pareto has the most path reliability, regarding the more importance considered for the safety in the samples.

3- More reliability requires more cost. For the data used in the present study, the most path reliability is equal to 0.710 and the least value of objective function of cost corresponding to the reliability of 0.710 was obtained with the discount factor of 0.2.

4- For the samples of the most discount ($\alpha = 0.2$), the selected hub number and total cost are the least and the reliability decreases. However, for the problems with the least discount ($\alpha = 0.8$), cost and reliability increase due to an increase in the hub number.

5- By increasing p value in calculations of capacity (relation (4.22) and fixed cost of establishing hubs (relations 4.23 and 4.24), the number of established hubs increases and the time required for solving problem decreases in most cases.

5 Conclusion

In this study, a bi-objective capacitated single-allocation hub location problem was examined assuming the reliability of paths. The first and second objects of this problem were to minimize the total cost of establishing hubs and transporting, and to maximize the reliability of the weakest path, respectively. To this aim, three mathematical models were introduced and their functions were investigated. Two methods based on capacity and the distance from center of mass were used for determining fixed cost of establishing hubs. The single-objective models aiming to minimize cost were studied and the bi-objective models were evaluated after determining the model and parameter of selected cost. The results show that among the single-objective models, the third model is the best for solving CAB sample. In fact, the results of solving these models demonstrate that calculating fixed cost based on distance from center of mass causes more decrease in objective function value. Therefore, the third bi-objective model with fixed cost calculated based on distance from center of mass was used for solving the bi-objective problem. The bi-objective problem was solved by ε-constraint method. Two evaluation indexes, WSM and AHP were used for determining more appropriate answer among the generated Pareto answer set. Using these two indexes can enhance flexibility of answers regarding the priorities of decision makers.

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