Multiquadric-Radial Basis Functions Method for Mortgage valuation under jump-diffusion model

Raheleh Jalili
Department of Applied Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran,
rahele.jalili@yahoo.com

Abdolsadeh Neisy
Department of Mathematics, Faculty of Mathematical Science, Allameh Tabataba'i University (ATU), Tehran, Iran,
(Corresponding Author)
a_neisy@atu.ac.ir,

Alireza Vahidi
Department of Mathematics, Shahr-e-rey branch, Islamic Azad University, Tehran, Iran, email: alrevahidi@yahoo.com

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ABSTRACT

Given the significant benefits of the Radial Basis Function (RBF) approach, here in this paper, we tried to exploit and adopt this method for the Fixed-Rate Mortgage (FRM) models. In the real world, a jump occurs due to an unknown reason and perhaps better reflects the evolution of real estate prices during bubbles and crises in the real estate markets. For the house price evolution, the jump-diffusion models are used which would lead to a Partial-Integro Differential Equation (PIDE) model. The main concentration is on the difficulty of projecting the pricing of FRM that deals with contracts in where the underlying stochastic factors are the house price and the interest rate. Utilizing the stochastic house-price and stochastic interest-rate models, we were able to develop a reliable mortgage valuation. The identified Partial-Integro Differential Equation (PIDE) from the FRM pricing model, solved by RBF considering the fact that a closed-form solution is usually unavailable. Further, to display the expected behavior of the contract, the possible applications of the suggested method applied to UK fixed-rate mortgages. Based on available resources, a set of economic parameters was determined for the mortgage to provide an instance to show the applicability of the proposed approach.

Keywords:

Fixed-Rate Mortgages, Jump-diffusion model, Partial-Integro Differential Equation, Stochastic interest rate, Radial Basis Function.

1. Introduction

A mortgage is a financial contract between two a borrower and a lender where the borrower uses a risky asset like a house (as collateral) and gains funds, usually from a bank or a financial institution. In the real world, a jump occurs due to an unknown reason and perhaps better reflects the evolution of real estate prices during bubbles and crises in the real estate markets. This study attempts to address the problem of estimating the pricing of a Fixed-Rate Mortgage (FRM) and deals with contracts in which the underlying stochastic factors are the house price and the interest rate. In this article, for the house price evolution, the jump-diffusion models are used. Utilizing the stochastic house-price and stochastic interest-rate models, we were able to develop a reliable mortgage valuation.

The geometric Brownian motion was used by the Black and Scholes (1973) to construct a model of option pricing. The Black and Scholes model is recognized as a constant volatility model whereas the other model, which was offered in the financial mathematics literature in the late eighties, is generally known as modifications of the classical Black-Scholes model. Several suggestions exist in the literature to model the stochastic interest rate, such as Cox, Ingersoll and Ross (1985), and Vasicek (1977). The focus herein is taking into consideration the Cox, Ingersoll and Ross (1985) model for the stochastic interest rate.

In the finance literature, several examples of the valuation of financial derivatives exist when the underlying assets follow a jump-diffusion process. In this study, we use the jump-diffusion models for the house price evolution, which would lead to a Partial-Integro Differential Equation (PIDE) model. Thus, it is the main innovative point of the present paper. Here we suppose that the house price dynamics is governed by Merton (1976) and Kou (2002) jump-diffusion models and we assume a finite number of jumps following a Poisson process. In the case of fixed rate mortgages, we obtain a sequence of PIDE problems (one for each month) in order to get the value of the mortgage. Concerning the numerical methods for solving PIDE problems arising in finance, Chan and Hubbert (2014) demonstrated an efficient numerical solution of the European and American option prices using the jump-diffusion model and Radial Basis Function (RBF) interpolation techniques. Here, the identified PIDEs from the FRM pricing model will solve by RBF, too.

The RBF interpolation scheme is a well-known meshless technique that has recently been used to solve PDEs in quantitative finance (for example, Fausshauer et al. (2004a), Fausshauer et al. (2004b), Hon and Mao (1999)). The new splitting scheme for solving a three-dimensional PDE was considered by Safaei et al. (2018), which divides each time step into fractional time steps with the more straightforward operator. In 1990, this numerical scheme was originally reported by Kansa (1990) for estimating partial derivatives using RBFs and resulted in a new method for solving partial differential equations. Considering RBFs' numerous advantages, this method has been implemented for various option pricing problems in Wilmott magazine (Pena, 2005) by using the Kansa (1990) collocation method. In Franke’s review paper (1982), Hardy’s multi quadratic (MQ) (1971) as one of the best interpolation methods is examined, and it is rated in terms of accuracy, speed, and ease of implementation. For instance, the MQ’s comparatively high accuracy has been made it favored choice in Pena (2005), Hon and Mao (1999), and Fausshauer et al. (2004b). Many different types of RBFs exist, and we can find more details in Liu (2003). In this case, MQ could possibly be one of the optimal algorithms for the scattered data interpolation problem. Given the significant benefits of the RBF approach (form MQ), here in this paper, we tried to exploit and adopt this method for the FRM models.

This paper's remainder is formed as follows: Section 2 describes the stochastic variables, provides a brief review of both the Merton and Kou jump-diffusion models, and derives the pricing models from estimating the FRM. Section 3 will look more closely at the RBF method and show how this technique may be used to discover the solution of PIDEs, which we then implement in the jump-diffusion model. In Section 4, the applicability of the proposed approach is indicated by a numerical example. Finally, in the fifth Section, the paper ends with conclusions and remarks.

Literature Review

Mortgage valuation models under jump-diffusion processes

To approximate the pricing of the Fixed-Rate Mortgage (FRM) we derive the pricing model, in which the underlying stochastic factors are the house price, H_t, and the interest rate, r_t. According to Merton (1976) and D'Halluin et al. (2005a), for example, the original form of the following Stochastic Differential Equation (SDE) with H_t for our market is considered

$$dH_t = (\mu - \delta)H_t \mathrm{d}t + \sigma_H H_t dZ_t^H + H_t d(\sum_{i=1}^{N_t}(V_i - 1)) \quad (1)$$

Our focus herein is taking into account the other source of uncertainty in stochastic volatility models. So it is assumed the interest rate r_t at the time t generated from the Cox, Ingersoll and Ross (1985) (CIR) process. CIR process is describing r_t as a mean-reverting square root process as following

$$dr_t = \kappa(\theta - r_t)dt + \sigma_r \sqrt{r_t} dZ_t^r. \quad (2)$$

Where the parameters indicate:
The instantaneous rate of house-price appreciation,
The house service flow,
The volatility of house price,
A standardized Wiener process of the house price,
The $exp(Y_i)$, Y_i is an i.i.d. sequence of random variables,
The mean reversion speed of r_t,
The long-run means of r_t-process,
The volatility of the r_t-process,
A standardized Wiener process of the interest rate,
The Poisson process with the Poisson arrival intensity $\tilde{\lambda}$.

Moreover, in Eq. (1) and (2) ${Z_t}^H$ and Z_t^r are standard Wiener processes having contact correlation ρ_{Hr} and η is a constant equal to $\eta = E[exp(Y_i)] - 1 = \int_0^\infty (e^v - 1)g(v)dv$ (Chan and Hubbert (2014)). The value of η is determined by $g(v)$, the probability density function of Y_i. Since the jump size has some known probability density $g(v)$, then $\int_0^\infty g(v)\,dv = \int_{-\infty}^\infty g(v)dv = 1$ and we consider only the positive jumps such that $g(v) = 0$ if $v < 0$. Now, $g(v)$ may have any appropriate probability density function. For example, Merton's model (1976) assumes $g(v)$ to be log-normal with $Y_i \sim N(\mu_j, \sigma_j^2)$ whereas Kou's model (2002) assumes it to be double exponential

$$\begin{cases} 1/\sqrt{2\pi}\sigma_j . e^{-(v-\mu_j)^2/2\sigma_j^2} & for the Merton model or \\ p.\alpha_1 e^{-\alpha_1 v} 1_{v\geq 0} + q.\alpha_2 e^{\alpha_2 v} 1_{v<0} & for the Kou model, \end{cases} \quad (3)$$

where p, q, α_1 and α_2 are positive constants such that $p + q = 1$ and $\alpha_1 > 1$. And also, the value for η for Merton or Kou models is

$$\eta = e^{\mu_j + (\sigma_j^2/2)} - 1 \quad \text{or} \quad \eta = (p\alpha_1/\alpha_1 - 1) + (q\alpha_2/\alpha_2 + 1) - 1,$$

respectively. Cont and Tankov (2004) explain more careful derivation of jump models with Lévy process and are not repeated here. Next section presents the model for the evolution of the price of the underlying assets when the underlying assets follow a jump-diffusion model for the house price evolution. Then it will show the general form of the Partial-Integro Differential Equation (PIDE) to be solved for FRM valuation.

PIDE Fixed-rate mortgage pricing formula under jump-diffusion processes

For the house price evolution, the jump-diffusion models are used which would lead to a PIDE model. Then standard techniques based on Itô (1951) formulas for jump-diffusion process prove that the valuation of any asset $F_t = F(t, H_t, r_t)$, whose value is a function only of house price H_t, interest rate r_t and time t, satisfies the following PIDE (see Cont and Tankov (2004), Calvo-Garrido and Vázquez (2015), for example):

$$\partial_t F + \frac{1}{2}\sigma_H^2 H^2 \partial_{HH} F + \rho\sigma_H\sigma_r H\sqrt{r}\partial_{Hr} + \frac{1}{2}\sigma_r^2 r \partial_{rr} F + (r - \delta)H\partial_H F + \kappa(\theta - r)\partial_r F - rF + \int_0^{+\infty} \tilde{\lambda}[F(t, H\, exp(v), r) - F(t, H, r) - H(exp(v) - 1)\partial_H F(t, H, r)]g(v)dv = 0. \quad (4)$$

In Eq. (4), partial derivatives are specified by sub-indexes in the symbol ∂ and henceforth, the dependence on t is suppressed to simplify notation. To specify the distribution of jump sizes, we will assume either Merton model (1976) or Kou model

(2002) with the density Eq. (3), respectively. Therefore, the PIDE in Eq. (4) can be written in the form

$$\partial_t F + \frac{1}{2}\sigma_H^2 H^2 \partial_{HH} F + \rho\sigma_H\sigma_r H\sqrt{r}\partial_{Hr} F + \frac{1}{2}\sigma_r^2 r\partial_{rr} F + (r - \delta - \tilde{\lambda}\eta)H\partial_H F + \kappa(\theta - r)\partial_r F - (r + \tilde{\lambda})F + \tilde{\lambda}\int_0^{+\infty} F(t, H\,exp(v), r)\,g(v)dv = 0$$
(5)

that models jump diffusion in FRM pricing for $0 \le \tau \le T$, $0 \le H < \infty$and $0 \le r < \infty$. More precisely, the functions defining the values of the mortgage $V(\tau_m, H, r)$ to the lender during the month m satisfy this PIDE as well. The remaining mortgage contract details, including the formulae for the monthly payment (MP), the outstanding balance following each payment $P(m-1)$, the usual monthly payment-date conditions, more details, and the boundary conditions may be found in Calvo-Garrido and Vázquez (2015), Sharp et al. (2008) and Azevedo-Pereira et al. (2002). The mortgage pricing problem starts from the value of the mortgage at maturity $t = T_M$, which just before the last payment is the minimum of MP and the house value, given by $V(\tau_M = 0, H, r) = min(MP, H)$. While at the other payment dates, it is provided by $V(\tau_m = 0, H, r) = min(V(\tau_{m+1} = T_{m+1}, H, r) + MP, H)$ where $1 \le m \le M - 1$ (Calvo-Garrido and Vázquez (2015)).

There is an integral term in the Eq. (5) due to the presence of jumps which makes the PIDE more difficult to solve (Calvo-Garrido and Vázquez (2015)). Considering the FRM pricing PIDE based on jump diffusion model derived in Eq. (5), the numerical procedure is not straight forward. It means that Eq. (5) has no closed form solution and we still try to solve this equation using RBF method. Therefore, we need some variable transformation before we proceed to solve Eq. (5). In addition, the variables that appear in the integral term need to be changed. Let $x = ln\,H$, $\tau = T - t$, and so that $F(t, H, r) = u(\tau, x, r)$. And also we get $\boldsymbol{F(t, He^v, r) = u(\tau, x + y, r)}$. Again, we change the variables in $\int_{-\infty}^{+\infty} u(\tau, x + y, r) f(y)dy$ to the new integral term, $z = x + y \rightarrow y = z - x$ and $dy = dz$. The partial integro differential Eq. (5) in terms of the new transformed variables x, z and τ will be

$$u_\tau = \frac{1}{2}\sigma_H^2 u_{xx} + \frac{1}{2}\sigma_r^2 r u_{rr} + \sigma_H\sigma_r\rho_{Hr}\sqrt{r}u_{xr} + (r - \delta - \frac{1}{2}\sigma_H^2 - \tilde{\lambda}\eta)u_x + \kappa(\theta - r)u_r - (r + \tilde{\lambda})u + \tilde{\lambda}\int_{-\infty}^{+\infty} u(\tau, z, r) f(z - x)dz. \quad (6)$$

In this part we have given the basic insight to the FRM pricing models. After finding the pricing models of FRM under the underlying asset the PIDE identifies, it is important to be able to find the solution of the pricing equation efficiently. Since the closed-form solution to the PIDE is primarily unavailable, they have to be solved numerically. Next part highlights the popular numerical method we wish to adopt to solve pricing problems.

Methodology

According to RBF's advantages with the fast development in many research fields over the last two decades, this method has been explored by Pena (2005) and Pettersson et al. (2008). Now we are able to show the RBF as a well-known meshless technique for reconstructing an unknown function from scattered data and for the pricing of financial contracts by solving the PIDE. To do this, one must first obtain an RBF approximation of the initial value of the contract. Using an RBF interpolant with the RBF scheme, we approximate the unknown function $u = u(\tau, x, r)$ to determine the interpolation points for the initial value. Then we derive a system for the linear constant coefficient ODE by requiring that the PIDE Eq. (6) be satisfied for the chosen RBF interpolation points. Due to the numerous RBF types, the readers can refer to Liu (2003). After selecting the interpolation points, we interpolant the unknown function u by the radial basis function $\phi(\|x - x_k, r - r_j\|)$ called RBFs and φ is a basic function

$$u(\tau, x, r) \cong \sum_{k=1}^{M}\sum_{j=1}^{N} c_{kj}(\tau)\phi(\|x - x_k, r - r_j\|), \quad (7)$$

where $c_{kj} \in R$ is estimated by the interpolation condition $u(x_k, r_j) =: \tilde{u}(x_k, r_j)$, and we use the Euclidean norm. In this research, to approximate the solution u, we utilize the RBF of form MQ which is a favored choice in the literature (Pena (2005), Fausshauer et al. (2004b), Hon and Mao (1999)), rather than the other popular basis functions. We get a linear equations system when we substitute the righthand side of Eq. (7) into Eq. (6). So we gain the matrix form $Ac = u$, where c shows the vectors containing the unknown coefficients $c = [c_1, \ldots, c_{MN}]^T$, and $A_{k'j'kj} = \phi(\|x_{k'} - x_k, r_{j'} - r_j\|)$, $k, k' = 1, \ldots, M, j, j' = 1, \ldots, N$. Because the RBF does not depend on time, the time derivative of u is simply the time derivatives of the coefficients. Therefore, the partial derivatives of $\tilde{u}$ concerning time and other underlying assets have to be found, respectively

$$\partial\tilde{u}(\tau, x_{k'}, r_{j'})/d\tau \approx \sum_{k=1}^{M}\sum_{j=1}^{N} dc_{kj}(\tau)/d\tau . \phi(\|x_{k'} - x_k, r_{j'} - r_j\|)$$

$$\partial\tilde{u}(\tau, x_{k'}, r_{j'})/\partial r \approx \sum_{k=1}^{M}\sum_{j=1}^{N} c_{kj}(\tau) . \partial\phi(\|x_{k'} - x_k, r_{j'} - r_j\|)/\partial r$$

$$\partial^2\tilde{u}(\tau, x_{k'}, r_{j'})/\partial r^2 \approx \sum_{k=1}^{M}\sum_{j=1}^{N} c_{kj}(\tau) . \partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|)/\partial r^2$$

$$\partial\tilde{u}(\tau, x_{k'}, r_{j'})/\partial x \approx \sum_{k=1}^{M}\sum_{j=1}^{N} c_{kj}(\tau) . \partial\phi(\|x_{k'} - x_k, r_{j'} - r_j\|)/\partial x$$

$$\partial^2\tilde{u}(\tau, x_{k'}, r_{j'})/\partial x^2 \approx \sum_{k=1}^{M}\sum_{j=1}^{N} c_{kj}(\tau). \partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|)/\partial x^2$$

$$\partial^2\tilde{u}(\tau, x_{k'}, r_{j'})/\partial x\partial r \approx \sum_{k=1}^{M}\sum_{j=1}^{N} c_{kj}(\tau) . \partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|)/\partial x\partial r.$$

Besides, the first and second partial derivatives of φ for the MQ radial basis function should be obtained with respect to the underlying assets r, and x, respectively

$$\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma) = \sqrt{\|x_{k'} - x_k, r_{j'} - r_j\|^2 + \varsigma^2}$$

$$\partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial r^2 = (\sqrt{\|x_{k'} - x_k, r_{j'} - r_j\|^2 + \varsigma^2})^{-1} - (r_{j'} - r_j)^2(\sqrt{\|x_{k'} - x_k, r_{j'} - r_j\|^2 + \varsigma^2})^{-3}$$

$$\partial\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial x = (x_{k'} - x_k).(\sqrt{\|x_{k'} - x_k, r_{j'} - r_j\|^2 + \varsigma^2})^{-1}$$

$$\partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial x^2 = (\sqrt{\|x_{k'} - x_k, r_{j'} - r_j\|^2 + \varsigma^2})^{-1} - (x_{k'} - x_k)^2(\sqrt{\|x_{k'} - x_k, r_{j'} - r_j\|^2 + \varsigma^2})^{-3}$$

$$\partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial x\partial r = -(x_{k'} - x_k)(r_{j'} - r_j).(\sqrt{\|x_{k'} - x_k, r_{j'} - r_j\|^2 + \varsigma^2})^{-3}$$

where ς is the shape parameter with the huge impact on accuracy of interpolation matrix. Then we collocate and substitute of the expansions for $\tilde{u}(\tau, x_{k'}, r_{j'})$, $\partial\tilde{u}(\tau, x_{k'}, r_{j'})/\partial\tau$, $\partial\tilde{u}(\tau, x_{k'}, r_{j'})/\partial r$, $\partial^2\tilde{u}(\tau, x_{k'}, r_{j'})/\partial r^2$, $\partial\tilde{u}(\tau, x_{k'}, r_{j'})/\partial x$, and $\partial^2\tilde{u}(\tau, x_{k'}, r_{j'})/\partial x^2$ into the FRM pricing model Eq. (6).

Transforming the PIDE to a system of ODEs using Radial Basis Function

Furthermore, for notational simplicity, we use A, A_r, A_{rr}, A_x, A_{xx}, and A_{xr} as the $NM \times NM$ matrices of $\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)$, $\partial\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial r$, $\partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial r^2$, $\partial\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial x$, $\partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial x^2$, and $\partial^2\phi(\|x_{k'} - x_k, r_{j'} - r_j\|, \varsigma)/\partial x\partial r$. They express the first and second partial derivatives of φ for the MQ radial basis function in relation to the underlying assets r and x, respectively. Using equally spacing method described in the literature (Hon and Mao (1999), Fausshauer et al. (2004a)), we determine an interval $[xmax_{min}$ and $[rmax_{min}$ for a given $M = 0,1,\ldots$ and $N = 0,1,\ldots$, we have $x_k := x_{min}$ and $r_j := r_{min}$ where $\Delta x = x_{\max} - x_{\min} / M$ and $\Delta r = rmin_{max}$. So we arrive at the following system of ODEs (Chan and Hubbert (2014))

$$A\dot{c} = \frac{1}{2}\sigma_H^2 A_{xx}c + \frac{1}{2}\sigma_r^2 r A_{rr}c + \sigma_H\sigma_r\rho_{Hr}\sqrt{r}A_{xr}c + (r - \delta - \frac{1}{2}\sigma_H^2 - \tilde{\lambda}\eta)A_x c + \kappa(\theta - r)A_r c - (r + \tilde{\lambda})Ac + \tilde{\lambda}(\int_{-\infty}^{+\infty} A(z)f(z - x)dz)c \qquad (8)$$

where $\dot{C}$ indicates $dc_{kj}(\tau)/d\tau$. For truncating the integrals from an infinite to finite computational range some researchers have provided different numerical techniques, see Briani et al. (2007), D'Halluin et al. (2005). Here we use Briani et al. (2007) numerical technique for truncating the integral domain of the PIDE Eq. (8) in both the Merton (1976) and Kou (2002) models. Let $\varepsilon > 0$, for selecting $[z_{-\varepsilon}, z_{\varepsilon}]$ for the set of points z in Merton case is as follows

$$z_{\varepsilon} = \sqrt{-2\sigma_j^2 \, log\left(\varepsilon\sigma_j\sqrt{2\pi}/2\right)} + \mu_j, \forall z \geq 0$$

$$z_{-\varepsilon} = -z_{\varepsilon}, \forall z < 0,$$

and in Kou model we have

$$z_{\varepsilon} = log(\,\varepsilon/p)/(1-\alpha_1), \forall z \geq 0$$

$$z_{-\varepsilon} = -\,log(\,\varepsilon/q)/(1-\alpha_2), \forall z < 0.$$

therefore transform Eq. (8) into

$$A\dot{c} = \frac{1}{2}\sigma_H^2 A_{xx}c + \frac{1}{2}\sigma_r^2 r A_{rr}c + \sigma_H\sigma_r\rho_{Hr}\sqrt{r}A_{xr}c + (r - \delta - \frac{1}{2}\sigma_H^2 - \tilde{\lambda}\eta)A_x c + \kappa(\theta - r)A_r c - (r + \tilde{\lambda})Ac + \tilde{\lambda}(\int_{z_{-\varepsilon}}^{z_{\varepsilon}} A(z)f(z-x)dz)c \qquad (9)$$

For the matrix of the integrals evolution in Eq. (9), the MATLAB is used (Shampine (2008)). Then, the integrals in Eq. (9) can be approximated by

$$\int_{z_{-\varepsilon}}^{z_{\varepsilon}} \boldsymbol{A(z)f(z-x)dz \approx I} \qquad (10)$$

We substitute the Eq. (10) into Eq. (9), which leads the new approximate equation

$$A\dot{c} = \frac{1}{2}\sigma_H^2 A_{xx}c + \frac{1}{2}\sigma_r^2 r A_{rr}c + \sigma_H\sigma_r\rho_{Hr}\sqrt{r}A_{xr}c + (r - \delta - \frac{1}{2}\sigma_H^2 - \tilde{\lambda}\eta)A_x c + \kappa(\theta - r)A_r c - (r + \tilde{\lambda})Ac + \tilde{\lambda}Ic \qquad (11)$$

The Eq. (11) can be written as follow (Fausshauer et al. (2007))

$$\boldsymbol{\dot{c}} = \left(\frac{1}{2}\boldsymbol{\sigma_H^2 A_{xx}} + \frac{1}{2}\boldsymbol{\sigma_r^2 r A_{rr}} + \boldsymbol{\sigma_H\sigma_r\rho_{Hr}\sqrt{r}A_{xr}} + (\boldsymbol{r} - \boldsymbol{\delta} - \frac{1}{2}\boldsymbol{\sigma_H^2} - \boldsymbol{\tilde{\lambda}\eta})\boldsymbol{A_x} + \boldsymbol{\kappa}(\boldsymbol{\theta} - \boldsymbol{r})\boldsymbol{A_r} - (\boldsymbol{r} + \boldsymbol{\tilde{\lambda}})\boldsymbol{A} + \boldsymbol{\tilde{\lambda}I} \right)\boldsymbol{A^{-1}c}$$

Moreover,

$$\boldsymbol{\dot{c} = \Omega c}, \qquad (12)$$

where Ω can also expressed as

$$\Omega = \left(\frac{1}{2}\sigma_H^2 A_{xx} + \frac{1}{2}\sigma_r^2 r A_{rr} + \sigma_H\sigma_r\rho_{Hr}\sqrt{r}A_{xr} + (r - \delta - \frac{1}{2}\sigma_H^2 - \tilde{\lambda}\eta)A_x + \kappa(\theta - r)A_r - (r + \tilde{\lambda})A + \tilde{\lambda}I \right)A^{-1}$$

We derived the RBF-based method for our pricing models of FRM under the underlying asset from what has already been obtained. Now, we will employ the numerical method for solving the ODE in Eq. (12). The coefficient vector $\boldsymbol{C}$ in the mentioned equations can be defined later with our preferred time integration scheme, RK4 method which is of fourth order. Here, no prior knowledge of RK4 scheme is required. The next part explains results that illustrate the proposed numerical technique's performance for the mortgage to show this contract's expected behavior. In the following numerical example, all of our numerical simulations will be performed using MATLAB.

Results

This section presents a set of parameters which is needed to be specified to obtain the solution to the FRM valuation problem. Following details are presented of the contract parameters, which are fixed: $\sigma_r = 7\%$, $\sigma_H = 10\%$, $\theta = 10\%$, $\kappa = 25\%$, $\delta = 7.5\%$, $\rho = 0$, $r_0 = 8\%$, $\tilde{\lambda} = 0.1$, $\mu_j = -0.1$, $\sigma_j = 0.45$, $p = 0.3445$, $\alpha_1 = 3.0465$, $\alpha_2 = 3.0775$. Mortgage contract parameters are: $H_{initial} = 100{,}000€$, $T = 300$ months, $c_0 = 10\%$. Therefore, the results from the RBF method to price FRM models could be found. A basic set of parameters have been chosen by parameters reported in the literature (see Sharp et al. (2008), Azevedo-Pereira et al. (2002) and Kau et al. (1995), for example). Finding the FRM valuation problem and illustrating all required computations are the main reasons to specify such parameters. Moreover, some parameters and spatial discretization are collected whose values are needed regarding the numerical methods employed to solve the problem: $H_\infty = 200{,}000€$, $r_\infty = 40\%$, $\Delta r = 0.1$, Time steps per month is 30. We utilize the numerical scheme which is proposed in Briani et al. (2007) and we find out a finite computational range for our global integral of the PIDE. Moreover, in terms of meshless interpolation methods, we use MQ as a basis function.

Figure 1 illustrates the values of the mortgage taking into account jumps for the house value. In Figure 1 we consider Merton jump-diffusion model for house price dynamics. As expected, in the presence of jumps the value of the contract is lower than without jumps. Note that the presence of jumps increases uncertainty in the house price, thus depreciating the mortgage price. We took into account the fixed parameters of the model as mentioned above.

Besides, we are going to present a figure which represents the price changes of the mortgage values versus house price changes in Merton model in different maturity times.

Figure 2 displays the FRM price functions taking into account jumps for the house value corresponding to the set of benchmark parameters when the loan term is 25 years, for the various values of T.

Hence, the numerical results emphasize that the RBF method can be used efficiently for the valuation of a FRM using the jump-diffusion model. Then the contract price decreases when we suppose jump-diffusion dynamics for the house value.

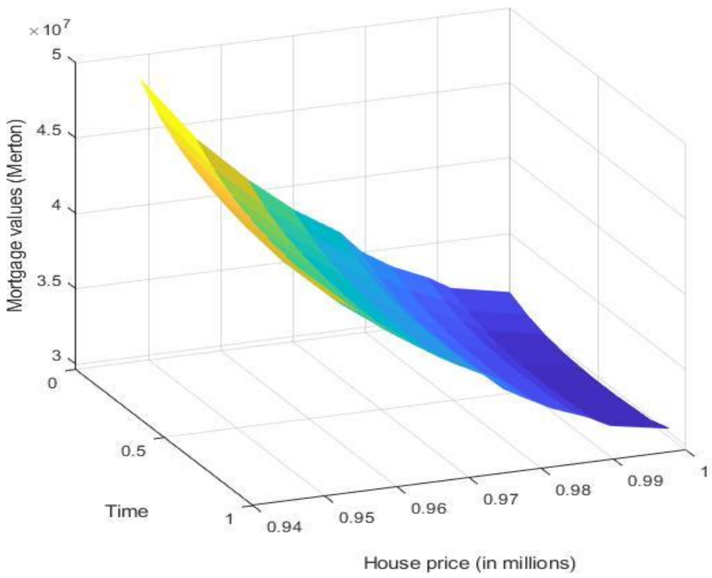

Figure 1. RBF approximation for the mortgage values under Merton jump-diffusion model for $r = 5\%$. Computions are based on the benchmark parameters, which are fixed and are taken from the literature mentioned above.

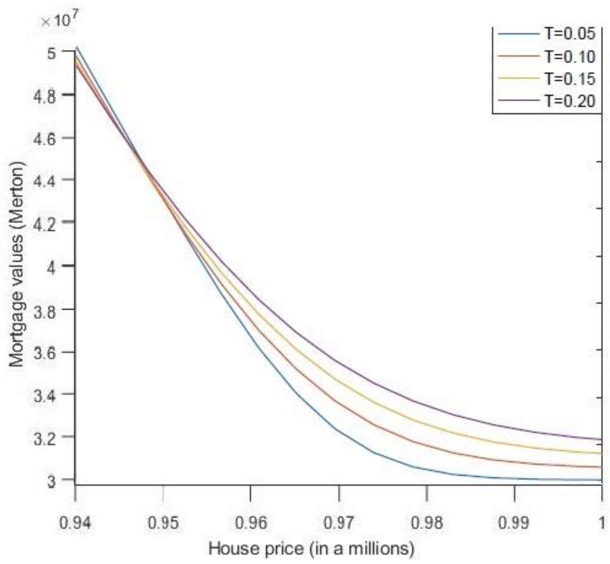

Figure 2. RBF approximation for the mortgage values under Merton jump-diffusion model for benchmark parameters and the various values of T.

Discussion and Conclusions

Given the numerous advantages of RBF, this method has been implemented for the valuation of a FRM using the jump-diffusion model. Mortgage valuation models based on stochastic house-price and interest-rate models were derived and where the house value is supposed to be driven by a jump-diffusion process. In reality, a jump occurs due to an unknown reason and perhaps better reflects the evolution of actual state prices in the financial crisis set in many countries. Hence, the assumption of jump-diffusion models instead of pure diffusion ones (GBM process) seems more reasonable. More precisely, we assumed that the jumps follow Merton (1976) and Kou (2002) models. We utilize the numerical scheme which is proposed in Briani et al. (2007) and we find out a finite computational range for our global integral of the PIDE. Moreover, in terms of meshless interpolation methods, we use MQ as a basis function. Finally, based on ODEs gained, we design the RK4 which is of fourth order. A basic set of economic parameters, based on literature, was specified for the mortgage as an example to demonstrate the potential application of the proposed approach. Besides, for one of the jump-diffusion models in the house price, we demonstrate some numerical results to illustrate the behavior of the methods. Finally, we include figure which represent the price changes of the mortgage values versus house price changes in Merton model. Also, we show the FRM price functions taking into account jumps for the house value which has been illustrated for different maturity times. In particular, if we suppose jump-diffusion dynamics for the house value then the contract price decreases, as expected.

Taking into account the interest rate, in this work, we include a figure for a fixed interest rate while it can be illustrated by many different ones. In terms of meshless interpolation methods, our method can be easily extended to other basic functions rather than MQ such as cubic spline. Given the fact that the information on financial market data is incomplete, as future work, the authors may aim to use an artificial neural network for estimating the parameters of the model.

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