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Congestion Calculation only by Solving a Linear Programming Model Through Fuzzy Data

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Abstract

The studies on congestion, in the field of either economic or operations research, have been grown in recent years. Calculating and determining congestion have two important aspects. By reducing the input in the decision-making unit, which has experienced congestion, we can also reduce its expenses. Moreover, congestion can lead to a reduction in outputs. By its obviation, output can be increased, and in turn, it can also increase the profits consequently. In this research, congestion of decision-making units in data envelopment analysis was estimated in the presence of fuzzy data. In most of the problems of the real and practical world, the obtained information is not exact, even they are fuzzy. Here, the interesting point is how to calculate congestion of data if they are fuzzy. At this point, congestion can be calculated through triangular fuzzy data using $\alpha - cut$ in a particular interval. The suggested model provides an interval solution, which can be used as a guide in decisions making.

Keywords: Data envelopment analysis; Congestion; Triangular fuzzy data; $\alpha - cut$

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1. Introduction

The principal objective of this study was suggesting a novel method to measure the efficacy and stocks rating [1]. Although, the role of Data Envelopment Analysis (DEA) models with fuzzy data is known to be a powerful tool for solving the real world issues, there are some limitations such as DEA sensitivity in the data. Since, DEA is a method concentrating on boundaries, a disorder or mistake in measuring the data can lead to major problems.

Therefore, to successfully use it, the input and output need to be accurately calculated. For The first time, the theory of fuzzy set was presented by Professor Lotfi-Askarzadeh in 1956. Nevertheless, in the research literature of fuzzy DEA, several fuzzy approaches can be found for evaluating the efficacy. Guo and Tanaka [2] considered the CCR model by symmetrical triangular fuzzy numbers. After using the $\alpha - cut$ of limitations and comparing the obtained results, they evaluated the efficacy of units by solving two problems related to linear planning. Another method in DEA was suggested by Saati et al. in 2002 [3].

The performance of decision-making units, which can be evaluated by DEA models, is affected by the number of resources that are usable. Usually, along with an increase in input resources, the outputs can increase as well. However, this is not accurate for all manufacturing technologies. Sometimes, increase in input resources can result in outputs reduction.

Investigating the congestion has been increasingly considered in recent times, either in economic or operations research. Many researchers have studied the phenomenon of congestion, and also presented various models for its evaluation. Congestion refers to positions in which a one or several inputs reduction, can result in one or several outputs increase.

For The first time, studying the congestion was performed by Fare and Svensson [4] in 1980. While before that, this subject was unidentified and ambiguous in the economy. In 1983, Far and Grosskopf [5] completed this subject and prepared a model according to DEA. This issue has led to increased research in this field; therefore, another method [6] was proposed in 1996. Kooper [7] studied congestion management in China industry, and suggested how the evaluation of managerial inefficiency can have led to increased output without causing a reduction in employment in knitting and automobile industry.

The issue of industry congestion was reviewed by Dr. Jahanshahloo and Khodabakhshi [8]. The proposed models provide the evaluation principles for DMUs. Tone and Sahoo [9] suggested a procedure, which is non-parametric in structural terms, and can be used for measuring elastic scale in production of congestion. Kooper [10] studied the cases of congestion through DEA randomly by planning. Odeck [11] studied the congestion effect on samples such as fuel consumption, and the total number of workers on the efficiency of the Norwegian bus industry.

The suggested models, regardless of the usual methods related to fuzzy DEA, have the ability of endogenously regulating the level of confidence belonged to each constraint, along with constructing, with respect to numerous fuzzy measures, both conservative and non-conservative methods [12]. One of the most applicable and widespread methods in the literature to manage inaccurate and vague data in DEA models is Possibilistic Data Envelopment Analysis (PDEA) [13].

Jahanshahloo and Khodabakhshi have presented a procedure based on the model of improving the output to calculate congestion. This model was prepared as two other models, and it required the solution of three linear programming problems. Khodabakhshi reduced this method in a form in which it required only one model to be solved, which is of great importance in terms of its calculation [14]. This aim of this study was to provide a method to calculate the congestion, based on triangular fuzzy data. Since the data are not exact in the real world, this method can help us solving the issues related to congestion problems in a fuzzy manner. In this method, we calculated congestion based on triangular fuzzy data, and also by using $\alpha - cut$ in a particular interval. The obtained interval can be used in decision making. In part 2, the input relaxation model is introduced. In section 3, one-model approach is presented. In section 4, fuzzy problems are explained. In section 5, our proposed method for calculating congestion through fuzzy data is presented. In section 6, an example is studied. In this example, the data are in the form of triangular fuzzy data, and congestion has been calculated in a particular interval. In the final section, the conclusion is presented.

2- The input relaxation model

Jahanshahloo and Khodabakhshi [15,16] have presented the following model to improve production:

$$\begin{aligned} & Maximize\, \varphi_0 + \varepsilon\left(\sum_{i=1}^{m} s_{i1}^- + \sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_{i2}^+\right) \\ & S.to \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_{i1}^- - s_{i2}^+ = x_{i0}, i = 1,...,m \\ & \qquad \sum_{j=1}^{n} \lambda_j y_{rj} - \varphi_0 y_{r0} - s_r^+ = 0, r = 1,...,s \\ & \qquad \sum_{j=1}^{n} \lambda_j = 1 \qquad\qquad (1) \\ & s_{i1}^-, s_{i2}^+, \lambda_j, s_r^+ \geq 0, i = 1,...,m;\ j = 1,...,n; r = 1,...,s. \end{aligned}$$

The current models in DEA show the change of input ratio is based on the reduction of outputs. The logical explanation for this issue, from an economic standpoint, is that a reduction in outputs leads to a reduction in expenses. But in some cases, a reduction in input such as labor force might face with social tension. Consequently, in order to increase outputs, it is necessary to determine a combination of inputs commensurate with the situation of the society. Consequently, it is necessary to think about some flexibility in order to determine a combination of consumed inputs to calculate the best output.

In model (1) of Jahanshahloo and Khodabakhshi, the objective function is in such a way that $S_{i1}^{+^*}$, $S_{i2}^{+^*}$ are maximized and minimized respectively, i.e. the maximum value possible is deducted from *i*th input or the minimum useful value is added to it because the consumption of extra input brings about some expenses. Because we are going to calculate the highest output.

Since

$\forall i s_{i1}^+ = 0, s_{i2}^- = 0, \lambda_0 = 1, \lambda_j = 0 (j \neq 0), \forall r s_r^+ = 0, \varphi_0 = 1$

is a feasible solution to the problem, then model (1) is always feasible?

Definition 1. DMU_0 is efficient under model (1) if the below two conditions are satisfied:

i) $\phi_0^* = 1$

ii) Optimal amounts of all slacks are zero [14].

3. Congestion

If an increase in inputs, leads to a reduction in outputs, then there is congestion in the input. For instance, the high miners number in an underground mine is an example in which the number reduction of miners can lead to an increase in the value of the mine. The exact definition of congestion in general terms is as the following.

Definition 2. The input congestion occurs when one or several inputs increase without improving some outputs or vice versa. Congestion can also occur when some inputs reduce, without increasing some outputs [14].

Definition 3. Technical inefficiency occurs when some inputs or outputs are improved without worsening other outputs or inputs [14].

An easy way for relating these definitions to each other is to consider technical inefficiency synonyms with waste. Refer to Jahanshahloo and Khodabakhshi [15] who have determined the input congestion by using input relaxation model in the second stage. Also see [17,18] and [19] which have used an optimal solution $(\varphi_0^*, \lambda^*, s_1^{-*}, s_2^{+*}, s^{+*})$ in the following model to determine the technical inefficacy of the inputs.

$$\begin{aligned} & Maximize \sum_{i=1}^{m} \delta_i^+ \\ S.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} - \delta_i^+ = (x_{i0} - s_{i1}^{-*} + s_{i2}^{+*}), i = 1,...,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} = \varphi_0^* y_{r0} + s_r^{+*}, \quad r = 1,...,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \qquad (2) \\ & \delta_i^+ \le s_{i1}^{-*}, \quad i = 1,...,m \\ & \delta_i^+, \lambda_j \ge 0, \quad i = 1,...,m; \ j = 1,...,n. \end{aligned}$$

In the end, the value of *i*th the input congestion has been defined as follows:

$$s_i^{-c} = s_{i1}^{-*} - \delta_i^{+*}, \quad i = 1,...,m \qquad (3)$$

3.1 Khodabakhshi 's A one-model approach

Consider model (1) and supposed to $(\varphi^*, \lambda^*, s_1^{-*}, s_2^{+*}, s^{+*})$ be an optimal solution for it. Consequently, the first part of the model (2) can be as follows:

$$\sum_{j=1}^{n} \lambda_j x_{ij} - \delta_i^+ = x_{i0} - s_{i1}^{-*} + s_{i2}^{+*}, \quad i = 1,...,m$$

By writing this part in the following way:

$$\sum_{j=1}^{n} \lambda_j x_{ij} + (s_{i1}^{-*} - \delta_i^+) = x_{i0} + s_{i2}^{+*}, \ i = 1,...,m$$

And using a change in the variable $s_i^{-c} = s_{i1}^{-*} - \delta_i^{+*}$ will look like the following:

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-c} = x_{i0} + s_{i2}^{+*} \qquad i = 1,...,m$$

Therefore, we can rewrite model (2) as follows:

$$\begin{aligned} & Maximize \sum_{i=1}^{m} -s_i^{-c} \qquad (4) \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = (x_{io} - s_i^{-c} + s_{i2}^{+*}), i = 1,...,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} = \phi_o^* y_{ro} + s_r^{+*}, r = 1,...,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & s_i^{-c} \ge 0, \quad i = 1,...,m \\ & \lambda_j \ge 0, \quad j = 1,...,n. \end{aligned}$$

Where $(\phi_0^*, s_{i1}^{-*}, s_{i2}^{-*}, s_r^{+*})$ is the optimal solution of model (2). In order to calculate model (5), models (4) and (1) should be combined together. Consequently, the following model can be acquired:

$$\begin{aligned} & Maximize \ \varphi_0 + \varepsilon(\sum_{i=1}^{m} -s_i^{-c} + \sum_{r=1}^{s} s_r - \sum_{i=1}^{m} s_{i2}^{+}) \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-c} - s_{i2}^{+} = x_{i0}, i = 1,...,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - \varphi_o y_{ro} - s_r^{+} = 0, r = 1,...,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \qquad (5) \\ & s_i^{-c}, s_{i2}^{+}, \lambda_j, s_r^{+} \ge 0, \quad i = 1,...,m; \ j = 1,...,n; r = 1,...,s. \end{aligned}$$

If $(\varphi_0^*, \lambda^*, s_1^{-c*}, s_2^{+*}, s^{+*})$ is an optimal answer to model (5), then, $(\phi^*, s_2^{+*}, s^{+*})$ are part of the optimal answer (1) and (λ^*, s_1^{-c*}) is an optimal answer to model (4). In other words, model (4) is a part of the two-staged model (5).

4. Fuzzy Data Envelopment Analysis Model

In 1965, the theory of fuzzy sets was first proposed by Professor LotfiAskarzadeh, who is an Iranian scientist and the professor of Berkey University of the United States. Many research has been performed on this theory, therefore the theory has significantly been developed in the recent time, and acquired various applications in various fields. The fuzzy set theory is about an action in uncertain conditions and represents many concepts and variables and systems mathematically, which are inaccurate and ambiguous. It is also able to provide the ground for induction, deduction, control, and decision-making in uncertain situations.

4.1 Fuzzy sets

If we convert {0 and 1} range to a closed [0,1] set, the classic set turns into a fuzzy set. Therefore, the fuzzy set A in U is defined as the bellow:

A: U →[0,1]

A(U) ∈[0,1]

A(U) is called the membership function and expressed the degree of membership of A to U.

4.2 $\alpha - cut$

The $\alpha - cut$ of the fuzzy set A, which is represented by A_α, is a non-fuzzy set and for each α in the closed [0,1] set is equal to:

$$A_\alpha = \{x \in U \mid A(x) \geq \alpha\}$$

4.3 Triangular fuzzy number

Triangular fuzzy number (l, m, u) is a fuzzy interval the membership function of which $A(x)$ is:

1. A continuous mapping from R to the closed interval [0, w], $1 \leq w < 0$.
2. Fixed and equal to zero in [-∞, l]
3. Strictly ascending in [l, m]
4. Strictly descending in [m, u]
5. Fixed and equal to zero in [u, +∞]

m represents central value and l and u represent a high and low range.

5. The suggested method

In real-world, decisions are made based on quantitative as well as qualitative data. It seems, therefore, that fuzzy theory is appropriate for such issues. It seems to be necessary to develop the theory for basic models based on data envelopment analysis for situations in which some or all inputs and outputs are inaccurate. We have considered the inputs, outputs, and slacks as triangular fuzzy numbers, hence our suggested model and method for calculating the congestion in such situations are as follows:

$$\begin{aligned}
Maximize \quad & \phi_0 + \varepsilon\Big(\sum_{i=1}^{m} -(s_i^{-c^l}, s_i^{-c^m}, s_i^{-c^u}) + \sum_{r=1}^{s}(s_r^{+l}, s_r^{+m}, s_r^{+u}) - \sum_{i=1}^{m}(s_{i2}^{+l}, s_{i2}^{+m}, s_{i2}^{+u})\Big) \\
S.t \quad & \sum_{j=1}^{n} \lambda_j (x_{ij}^l, x_{ij}^m, x_{ij}^u) + (s_i^{-c^l}, s_i^{-c^m}, s_i^{-c^u}) - (s_{i2}^{+l}, s_{i2}^{+m}, s_{i2}^{+u}) = (x_{i0}^l, x_{i0}^m, x_{i0}^u), \quad i = 1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j (y_{rj}^l, y_{rj}^m, y_{rj}^u) - \phi_0 (y_{r0}^l, y_{r0}^m, y_{r0}^u) - (s_r^{+l}, s_r^{+m}, s_r^{+u}) = 0, \quad r = 1,\dots,s \qquad (6) \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& (s_i^{-c^l}, s_i^{-c^m}, s_i^{-c^u}), (s_{i2}^{+l}, s_{i2}^{+m}, s_{i2}^{+u}), (s_r^{+l}, s_r^{+m}, s_r^{+u}), \lambda_j \geq 0, i = 1,\dots,m; r = 1,\dots,s; j = 1,\dots,n
\end{aligned}$$

$$Maximize\,\varphi_0+\varepsilon\left(\begin{array}{l}\sum_{i=1}^{m}-\left(\alpha s_i^{-c^m}+(1-\alpha)s_i^{-c^l},\alpha s_i^{-c^m}+(1-\alpha)s_i^{-c^u}\right)+\\ \sum_{r=1}^{s}\left(\alpha s_r^{+m}+(1-\alpha)s_r^{+l},\alpha s_r^{+m}+(1-\alpha)s_r^{+u}\right)-\sum_{i=1}^{m}\left(\alpha s_{i2}^{+m}+(1-\alpha)s_{i2}^{+l},\alpha s_{i2}^{+m}+(1-\alpha)s_{i2}^{+u}\right)\end{array}\right)$$

$$\begin{aligned}S.t\quad &\sum_{j=1}^{n}\left(\alpha x_{ij}^m+(1-\alpha)x_{ij}^l,\alpha x_{ij}^m+(1-\alpha)x_{ij}^u\right)\lambda_j+\left(\alpha s_i^{-c^m}+(1-\alpha)s_i^{-c^l},\alpha s_i^{-c^m}+(1-\alpha)s_i^{-c^u}\right)\\ &\left(\alpha s_{i2}^{+m}+(1-\alpha)s_{i2}^{+l},\alpha s_{i2}^{+m}+(1-\alpha)s_{i2}^{+u}\right)=\left(\alpha x_{i0}^m+(1-\alpha)x_{i0}^l,\alpha x_{i0}^m+(1-\alpha)x_{i0}^u\right)\\ &\sum_{j=1}^{n}\left(\alpha y_{rj}^m+(1-\alpha)y_{rj}^l,\alpha y_{rj}^m+(1-\alpha)y_{rj}^u\right)\lambda_j-\varphi_0\left(\alpha y_{r0}^m+(1-\alpha)y_{r0}^l,\alpha y_{r0}^m+(1-\alpha)y_{r0}^u\right)-\\ &\left(\alpha s_r^{+m}+(1-\alpha)s_r^{+l},\alpha s_r^{+m}+(1-\alpha)s_r^{+u}\right)=0\\ &\sum_{j=1}^{n}\lambda_j=1\\ &\left(s_i^{-c^l},s_i^{-c^m},s_i^{-c^u}\right),\left(s_{i2}^{+l},s_{i2}^{+m},s_{i2}^{+u}\right),\left(s_r^{+l},s_r^{+m},s_r^{+u}\right),\lambda_j\geq 0\end{aligned}\tag{7}$$

$$\begin{aligned}&Maximize\,\varphi_0+\varepsilon\left(\sum_{i=1}^{m}-\left(\alpha s_i^{-c^m}+(1-\alpha)s_i^{-c^l}\right)+\sum_{r=1}^{s}\left(\alpha s_r^{+m}+(1-\alpha)s_r^{+u}\right)-\sum_{i=1}^{m}\left(\alpha s_{i2}^{+m}+(1-\alpha)s_{i2}^{+l}\right)\right)\\ &s.t\sum_{j=1}^{n}\left(\alpha x_{ij}^m+(1-\alpha)x_{ij}^l\right)\lambda_j+\left(\alpha s_i^{-c^m}+(1-\alpha)s_i^{-c^l}\right)+\left(\alpha s_{i2}^{+m}+(1-\alpha)s_{i2}^{+l}\right)=\left(\alpha x_{i0}^m+(1-\alpha)x_{i0}^l\right)\\ &\sum_{j=1}^{n}\left(\alpha y_{rj}^m+(1-\alpha)y_{rj}^l\right)\lambda_j-\varphi_0\left(\alpha y_{r0}^m+(1-\alpha)y_{r0}^u\right)-\left(\alpha s_r^{+m}+(1-\alpha)s_r^{+u}\right)=0\\ &\sum_{j=1}^{n}\lambda_j=1\\ &\left(s_i^{-c^l},s_i^{-c^m}\right),\left(s_{i2}^{+l},s_{i2}^{+m}\right),\left(s_r^{+m},s_r^{+u}\right),\lambda_j\geq 0,i=1,\ldots,m;r=1,\ldots,s;j=1,\ldots,n\end{aligned}\tag{8}$$

$$\begin{aligned}&Maximize\,\varphi_0+\varepsilon\left(\sum_{i=1}^{m}-\left(\alpha s_i^{-c^m}+(1-\alpha)s_i^{-c^u}\right)+\sum_{r=1}^{s}\left(\alpha s_r^{+m}+(1-\alpha)s_r^{+l}\right)-\sum_{i=1}^{m}\left(\alpha s_{i2}^{+m}+(1-\alpha)s_{i2}^{+u}\right)\right)\\ &s.t\sum_{j=1}^{n}\left(\alpha x_{ij}^m+(1-\alpha)x_{ij}^u\right)\lambda_j+\left(\alpha s_i^{-c^m}+(1-\alpha)s_i^{-c^u}\right)+\left(\alpha s_{i2}^{+m}+(1-\alpha)s_{i2}^{+u}\right)=\left(\alpha x_{i0}^m+(1-\alpha)x_{i0}^u\right)\\ &\sum_{j=1}^{n}\left(\alpha y_{rj}^m+(1-\alpha)y_{rj}^u\right)\lambda_j-\varphi_0\left(\alpha y_{r0}^m+(1-\alpha)y_{r0}^l\right)-\left(\alpha s_r^{+m}+(1-\alpha)s_r^{+l}\right)=0\\ &\sum_{j=1}^{n}\lambda_j=1\\ &\left(s_i^{-c^m},s_i^{-c^u}\right),\left(s_{i2}^{+m},s_{i2}^{+u}\right),\left(s_r^{+l},s_r^{+m}\right),\lambda_j\geq 0,i=1,\ldots,m;r=1,\ldots,s;j=1,\ldots,n\end{aligned}\tag{9}$$

By calculating $\alpha-cut$ of objective function and limitations, we canalso obtain linear planning model as follows:
We need to calculate the congestion in low radius. In fact, DMUs is in the best situation but the rest of DMUs are in the worst situation possible.

We calculate the congestion in high radius. In fact, DMUs is in the worst situation and the rest of DMUs are in the best situation possible.

Example 1: In this example, we try to calculate the congestion for 10 DMUs [20]:
The results of the calculations of model (8) are presented in Tables (1), (2), and (3). The first column represents the name of DMU. The second and third columns represent the input and output values in the triangular fuzzy form. The fourth column represents the value. The fifth column represents the amount of input congestion in low radius. And the final two columns represent input slack in low radius and output slack in high radius, respectively.

Table1: The obtained values of the suggested model (8) for α = 0.5

DMU	I	O	φ_0^*	s_i^{-c}	s_{i2}^{+}	s_r^{+}
A	(1,2,3)	(12,14,16)	0.86	(0,0)	(0,0)	(0,0)
B	(2,3,4)	(13,14,15)	0.9	(0,2)	(0,0)	(0,0)
C	(1.5,3,4.5)	(10,10,10)	0.66	(0,2.5)	(0,0)	(0,0)
D	(3,4,5)	(10,11,12)	0.7	(0,4)	(0,0)	(0,0)
E	(2,2,2)	(8,10,12)	0.66	(0,0)	(0,0)	(0,0)
F	(3,3,3)	(9,12,15)	0.7	(0,1)	(0,0)	(0,0)
G	(1,1,1)	(6,9,12)	0.71	(0,0)	(0,0)	(0,0)
H	(1,1,1)	(9,10,11)	0.9	(0,0)	(0,0)	(0,0)
K	(3,4,5)	(5,6,7)	0.36	(0,4)	(0,0)	(0,0)
M	(4,4.5,5)	(9,9,9)	0.6	(0,4.5)	(0,0)	(0,0)

Table2: The obtained values of the suggested model (8) for α = 0.7

DMU	I	O	φ_0^*	s_i^{-c}	s_{i2}^{+}	s_r^{+}
A	(1,2,3)	(12,14,16)	0.92	(0,0)	(0,0)	(0,0)
B	(2,3,4)	(13,14,15)	0.94	(0,3.33)	(0,0)	(0,0)
C	(1.5,3,4.5)	(10,10,10)	0.68	(0,3.83)	(0,0)	(0,0)
D	(3,4,5)	(10,11,12)	0.73	(0,6.66)	(0,0)	(0,0)
E	(2,2,2)	(8,10,12)	0.69	(0,0)	(0,0)	(0,0)
F	(3,3,3)	(9,12,15)	0.76	(0,2.33)	(0,0)	(0,0)
G	(1,1,1)	(6,9,12)	0.78	(0,0)	(0,0)	(0,0)
H	(1,1,1)	(9,10,11)	0.94	(0,0)	(0,0)	(0,0)
K	(3,4,5)	(5,6,7)	0.39	(0,6.66)	(0,0)	(0,0)
M	(4,4.5,5)	(9,9,9)	0.61	(0,7.83)	(0,0)	(0,0)

Table3: The obtained values of the suggested model (8) for α = 0.9

DMU	I	O	φ_0^*	s_i^{-c}	s_{i2}^{+}	s_r^{+}
A	(1,2,3)	(12,14,16)	0.98	(0,0)	(0,0)	(0,0)
B	(2,3,4)	(13,14,15)	0.98	(0,10)	(0,0)	(0,0)
C	(1.5,3,4.5)	(10,10,10)	0.7	(0,10.5)	(0,0)	(0,0)
D	(3,4,5)	(10,11,12)	0.76	(0,20)	(0,0)	(0,0)
E	(2,2,2)	(8,10,12)	0.7	(0,0)	(0,0)	(0,0)

F	(3,3,3)	(9,12,15)	0.82	(0,9)	(0,0)	(0,0)
G	(1,1,1)	(6,9,12)	0.9	(0,0)	(0,0)	(0,0)
H	(1,1,1)	(9,10,11)	0.98	(0,0)	(0,0)	(0,0)
K	(3,4,5)	(5,6,7)	0.41	(0,20)	(0,0)	(0,0)
M	(4,4.5,5)	(9,9,9)	0.63	(0,24.5)	(0,0)	(0,0)

Now, we calculate congestion in high radius:

Table4: The obtained values of the suggested model (9) for α = 0.5

DMU	I	O	φ_0^*	s_i^{-c}	s_{i2}^{+}	s_r^{+}
A	(1,2,3)	(12,14,16)	1.16	(0,24.5)	(0,0)	(0,0)
B	(2,3,4)	(13,14,15)	1.07	(0,24.5)	(0,0)	(0,0)
C	(1.5,3,4.5)	(10,10,10)	0.75	(0,24.5)	(0,0)	(0,0)
D	(3,4,5)	(10,11,12)	0.85	(2,24.5)	(0,0)	(0,0)
E	(2,2,2)	(8,10,12)	0.83	(0,24.5)	(0,0)	(0,0)
F	(3,3,3)	(9,12,15)	1	(1,24.5)	(0,0)	(0,0)
G	(1,1,1)	(6,9,12)	1.11	(0,24.5)	(0,0)	(0,0)
H	(1,1,1)	(9,10,11)	1.11	(0,24.5)	(0,0)	(0,0)
K	(3,4,5)	(5,6,7)	0.48	(2,24.5)	(0,0)	(0,0)
M	(4,4.5,5)	(9,9,9)	0.66	(3.5,24.5)	(0,0)	(0,0)

Table5: The obtained values of the suggested model (9) for α = 0.7:

DMU	I	O	φ_0^*	s_i^{-c}	s_{i2}^{+}	s_r^{+}
A	(1,2,3)	(12,14,16)	1.09	(0,24.5)	(0,0)	(0,0)
B	(2,3,4)	(13,14,15)	1.05	(0,24.5)	(0,0)	(0,0)
C	(1.5,3,4.5)	(10,10,10)	0.73	(0,24.5)	(0,0)	(0,0)
D	(3,4,5)	(10,11,12)	0.82	(1.42,24.5)	(0,0)	(0,0)
E	(2,2,2)	(8,10,12)	0.78	(0,24.5)	(0,0)	(0,0)
F	(3,3,3)	(9,12,15)	0.94	(1.42,24.5)	(0,0)	(0,0)
G	(1,1,1)	(6,9,12)	1.03	(0,24.5)	(0,0)	(0,0)
H	(1,1,1)	(9,10,11)	1.06	(0,24.5)	(0,0)	(0,0)
K	(3,4,5)	(5,6,7)	0.46	(1.42,24.5)	(0,0)	(0,0)
M	(4,4.5,5)	(9,9,9)	0.65	(2.35,24.5)	(0,0)	(0,0)

Table6: The obtained values of the suggested model (9) for α = 0.9:

DMU	I	O	φ_0^*	s_i^{-c}	s_{i2}^{+}	s_r^{+}
A	(1,2,3)	(12,14,16)	1.03	(0,24.5)	(0,0)	(0,0)
B	(2,3,4)	(13,14,15)	1.02	(0,24.5)	(0,0)	(0,0)
C	(1.5,3,4.5)	(10,10,10)	0.72	(24.5,1.42)	(0,0)	(0,0)

D	(3,4,5)	(10,11,12)	0.8	(0,24.5)	(0,0)	(0,0)
E	(2,2,2)	(8,10,12)	0.74	(0,0)	(0,0)	(0,0)
F	(3,3,3)	(9,12,15)	0.88	(0,24.5)	(0,0)	(0,0)
G	(1,1,1)	(6,9,12)	0.94	(0,0)	(0,0)	(0,0)
H	(1,1,1)	(9,10,11)	1.02	(0,24.5)	(0,0)	(0,0)
K	(3,4,5)	(5,6,7)	0.44	(0,0)	(0,0)	(0,0)
M	(4,4.5,5)	(9,9,9)	0.64	(0,24.4)	(0,0)	(0,0)

The results of the calculations of model (9) are presented in Tables (4), (5), and (6) which show the amount of congestion in high radius.

6. Conclusion:

The present paper studied data envelopment analysis through fuzzy data. In the real world, the data are not accurate. Therefore, a model was presented to calculate congestion. All fuzzy problems in DEA can be solved through using the available methods including $\alpha-cut$ method and converting them to interval and linear problems. In most of these methods, the intervals on both sides of limitations are compared with each other by the $\alpha-cut$. Also, some methods have been provided to compare the intervals. Recently, instead of comparing the intervals, there has been a tendency to define a variable in each interval, which respects the limitations and optimizes the objective function. One of the advantages of the method applied in this paper is the acquisition of solution interval which can be used as a guide in decision making. According to the presented example, DMU (A) does not have a congestion in low radius but it has a congestion in high radius. It suggests that DMU (A) does not have a congestion in the best situation, but in the worst situation, the level of congestion reaches (0, 24.5). In the end, it can be recommended that more studies should be performed on the suggested model by using parametric data.

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