Evaluation of Parallel Market's Long-term Memory Based on DFA and ARDL-Based Detrending
(Case Study: Stock Market and Exchange Rate)

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ABSTRACT
In this study, the relationship between stock market long-term memory and exchange rate was studied. For this purpose, the analysis of detrended fluctuations was used and in order to detrend the data, two common detrending methods and cross-detrending were used. The research data included daily information of the stock market index and the dollar exchange rate during the period 2014/03/25 to 2021/02/07 and the data analysis was performed using the regression models. The results showed that the cross-trending of parallel markets produces different results in estimating the long-term memory of the data. According to the research findings, the stock index has a short-term memory under the conventional detrending method, while the cross-detrending method shows long-term memory for this index. The results for the exchange rate showed that under the conventional detrending method, the long-term memory of the exchange rate cannot be estimated in all market volatilities situations, while the cross-detrending method showed that the exchange rate loses its long-term memory in the face of increasing market fluctuations. The results also showed that under the cross-trending method, there is a direct and significant relationship between the long-term memory of the stock market index and the exchange rate.

Keywords:
Detrended fluctuations, Parallel markets, Cross detrending, long-term memory.

1. Introduction

Financial and economic growth literature have recognized the effective role of capital markets in ensuring long-term economic growth and sustainable development. Higher accumulation and mobilization of internal capital, allocating capital to productive sectors, transferring risk, and sharing and discovering prices are measures which explain the mentioned relationship. A few studies conclude a causal relationship between financial development and economic growth, while others support no causal relationship (Marcus et al., 2013).

However, empirical researches show a positive one-way relationship between capital market development, especially the stock market, and economic growth. Having shallow financial depth, capital markets play a limited but sustainable role in economic growth. In developing countries, the potential role of capital in economic development depends on the degree of capital market development. It has been proven capital market development in developed markets is significantly related to the degree of market power (Aktan et al., 2019).Emerging and less developed markets are usually less efficient than developed markets. Market participants bears higher transaction costs in less developed markets (Tokich et al., 2018). Market efficiency hypothesis plays a key role in the relationship between capital market development and other sectors of the macro economy.

The efficient market hypothesis (Fama, 1970) offers three versions: 1) weak, indicating there is no analysis of past market behavior (technical or fundamental), preventing market participants to obtain abnormal returns; this is because all the data of past price is instantly calculated at market price. 2) The semi-strong, indicating market value of a financial asset is updated and adjusted almost immediately to absorb new public information (market and non-market). The distance is equal to a random walk from previous prices. ; 3) Strong, assuming that even private information (insider information, not being published for a particular group) allows investors to make abnormal profits. The presence of memory in historical price data signals inefficiency in capital market. Price memory is classified into two types of short-term and long-term. However, it should be noted that long-term memory and fractal properties can also be considered equivalent. Since the existence of fractal properties indicates the existence of long-term memory (Ferreira, 2018). The degree of efficiency or inefficiency (presence of memory in data) of emerging markets date back to Dragota and Tilica (2014) and Neuronabi (2012). These studies examine the weak efficient market theory (data required for strong or semi-strong efficiency were probably not available in most cases) and fail to provide a clear explanation on the efficiency of capital markets. There are mixed evidences on the efficiency of capital markets, caused by time-varying nature of capital markets (Gabner and Moses, 2019).

Several traditional tests are applied to analysis the presence of long-term memory in data or market efficiency including Jarque-Bra test, to measure the normality of the distribution, heavy tailed return distribution and lack of memory in the data, parametric correlation test to measure the dependence of successive returns, non-parametric test of two to examine the randomness of the sequence of positive and negative returns, variance ratio test to determine the presence of non-correlated changes in data, unit root test to evaluate the reliability of time series data, or GARCH heterogeneous variance models to analyze seasonal patterns: January effect, month, week and *etc*. However, it may be possible to attribute mixed results to different evaluation methods. However, what stands out more is the close connection between financial markets and parallel capital markets. Capital market is affected by exchange rate fluctuations and these two markets are expected to have at least a causal relationship. Therefore, it can be expected that the information available in each of these markets also incorporates information from the parallel market. This is where the idea of extracting information from a market using information contained in the parallel market comes to mind. Detrended fluctuation analysis has been more applied to measure market memory and efficiency. This method uses under studied market information to analyze weak efficiency (lack of memory). This study shows information available in parallel markets can be used to measure market memory and market efficiency. Therefore, the detrending process in detrended fluctuations analysis is based on the cumulative relationship between stock index information and exchange rate data. Data trend is removed by considering the hidden information in the parallel market and then the remaining fluctuations are analyzed. The main research question is whether the detrended fluctuation analysis based on parallel market information provides different results comparing traditional detrending method? This research has contribution to previous researches in two cases. First, the analysis of de-trended fluctuations has been considered in many studies and several researchers have discussed this issue, while the basis of this analysis is the method of de-trending in the data. In this research, we offer another method for data de-trending that is based on conditional information of capital markets from each other. This study also investigates the relationship between long-term memory in parallel capital markets, while so far this relationship has not been considered by other researchers.

2. Theoretical framework and literature review

Exchange rates and stock market index have always been studied as two determinants of economic development. However, previous related studies have shown conflicting empirical evidence for the relationship between the two variables. Some studies have suggested there is a long-term equilibrium relationship between stock price index and exchange rate (Olaini et al, 2020). Other studies have shown this relationship is short-term (Bahmani Oskooi and Sohrabian, 1992). Some studies have not been able to provide evidence supporting the relationship between these two variables (Chir et al., 2020). In addition, there are controversies even in studies supporting the relation between stock markets and exchange rates. Some suggest stock price index and exchange rate are positively correlated, while others think the correlation should be negative. In general, the literature on the theoretical foundations of the relationship between stock markets and foreign exchange can be divided into two directions: the effect of international trade (Agroval, 1981) and the effect of portfolio equilibrium (Bahmani Oskooi and Sohrabian, 1992).

Agrawal (1981) believes the relationship between these two markets stems from the impact of international trade. Exchange rate fluctuations cannot directly affect stock prices of multinational corporations and exports. It can also indirectly affect domestic companies. For a multinational corporation, exchange rate fluctuations immediately affect the value of its foreign operations and continuously affect the firm's profitability. Domestic companies are also affected by exchange rate fluctuations, as they may still import inputs and export their products. When the exchange rate falls, export competition and, in turn, import costs increase. Thus, exchange rate depreciation has a positive (negative) effect on export (import) companies and increases their stock prices while an increase in exchange value has a negative (positive) effect on export (import) companies and reduces their stock prices (Zhai et al., 2020).

Contrary to Aguarwal (1981), Bahmani Oskooi and Sohrabian (1992) use portfolio equilibrium method to analyze the relationship between stock prices and exchange rates. They argue if stock value is effected by an external parameter, domestic investors' wealth increases. Following investment portfolio equilibrium theory, the demand for foreign exchange increases as well. It is followed by increased demand for money and raised interest rates. Foreign capital is absorbed and currency value increases. Therefore, if investors are optimistic about a country's stock market, foreign investors may increase their investment in the country's stock market due to speculative demand and it indirectly increases country's currency value.

Based on the view that the foreign exchange and stock markets, regardless of their relationship, have information content, we detrend data based on the information contained in the data itself and the information hidden in the other market. However, this approach has not been considered by researchers and they are more interested in traditional approaches. Relying on the relationship and impact of parallel markets on each other, this study presents a method to detrend data and analyze detrended fluctuations. The presented method can be effective in estimating long-term memory and fractal properties of capital markets. Few researches links memory and capital markets efficiency. For example, Hosseini Ara (2014) investigate the effects of asymmetry and long-term memory on conditional fluctuations between the real exchange rate and stock returns. The results indicate there are conditional fluctuations between the returns of foreign exchange and capital markets in Iran. These conditional fluctuations indicate the turbulence transmission between these markets, their interdependence and capital outflow between these markets by transmitting shocks and different domestic and foreign economic policies. Therefore, if there is risk and reduced returns in the capital market, capital is transferred to the foreign exchange market.

Bastan (2017) has studied the dynamic conditional correlation between oil prices and exchange rates in Iran with emphasis on long-term memory using FIEGARCH-DCC method. This study applies nonlinear dynamic conditional correlation models to investigate the relationships between two variables of oil price and exchange rate. In examining the relationship between conditional correlation between exchange rate and oil market, important features such as long-term memory and asymmetry are considered and the results supports the existence of long-term memory between exchange market and oil market fluctuations. Falahati (2019) examines the effect of exchange rate fluctuations on the return of the Tehran Stock Exchange. This study applies EGARCH model to investigate the effect of exchange rate fluctuations on stock returns. The results show there is a positive and significant relationship between dynamic prices of the two variables, *i.e.* exchange rate and stock market returns. The effect of asymmetric fluctuations between these two markets is not stable and fluctuates over time. The exchange rate is a key variable that can affect the internal and external balance of the national economy in an open economic environment, and the stock market is able to quickly reflect the exact changes in the real economy.

Zhang et al. (2019) analyze the multi-fractal properties of the Bitcoin market based on the de-trended fluctuations analysis. The results show both price and volume of Bitcoin market have multi-fractal behaviors, while small fluctuations are less fractal. The results also show long-term price correlation is stable for all fluctuations, while volumes are not. The findings also show there is a multi-fractal relationship between Bitcoin and Gold markets. Manif et al. (2020) examine behavioral biases in capital market by providing a measure of the similarity of returns on cryptocurrencies during the Covid-19 period. The main purpose of this study is to investigate the efficiency of cryptocurrency before and after Corona virus outbreak using multi-fractal analysis. The results show Covid 19 has a positive effect on the efficiency of cryptocurrency market. Yao et al. (2020) analyze multi-fractal nature of Brent crude oil prices and stock market. The results show both variables have a long-term continuous trend and there is a long-term correlation in stock and oil data. The results indicate NASDAQ index and oil prices have the strongest correlation and the stock market has the most multi-fractal nature. Jia et al. (2020) examine the impact of US-China financial friction on soybean future markets in US and China. The results show the correlation coefficients decrease significantly during trade friction and different correlation trends are found. The generalized Hurst power is significantly increased in all data, especially for large fluctuations. The results show major reduction in multi-fraction of these markets is explained by the effect of wide-tail efficiency distribution. Grivel et al. (2021) estimated short-term memory based on Hurst exponent and de-trended fluctuations analysis. In this research, using the matrix formula and without approximation, first the behavior of DFA and its higher order type are analyzed according to the number of available samples. For this purpose, data with short-term memory that can be modeled by a white noise, a moving average process and a random process and its autocorrelation value is reduced exponentially is used. The results of this study show that the short-term memory of the data can be estimated using fluctuation analysis and accurate methods. Mensi et al. (2021) in a study examined the effect of oil price volatility on long-term memory and weak efficiency of oil companies in the stock market. The results show evidence of an asymmetric multifractal nature for all markets. Moreover, the multifractality is stronger in the upward movement of the market returns. The degree of efficiency of the stock markets is shown to be time-varying and experienced a decrease during the 2008 global financial crisis (GFC), but an upside trend occurred during the recent oil price crash followed a significant decline during COVID-19. Ding et al. (2021) investigated the relations between Hurst exponent and fractional differencing parameter for long memory. By using the Monte Carlo simulation and empirical examinations, the results show that there is a distinct linear relationship between the Hurst exponent and the fractional differencing parameter. This linear relationship may provide an efficient estimation of the range of Hurst exponent and fractional differencing parameter with each other when the stable index of the stable distribution is close to 2. Tiwari et al. (2021) examined the existence of long memory in crude oil and petroleum products. The results indicate that the weak-form efficiency in energy spot markets is clearly time-varying, with USGC(U.S. Gulf Coast Conventional Gasoline) Diesel Fuel the most efficient and Propane the least. Gómez-

Gómez et al. (2021) have examined the multifractal detrended fluctuation analysis of temperature in Spain. Outcomes corroborate that all temperature variables have multifractal nature and show changes in multifractal properties between different periods. Also, Hurst exponent's values indicate that all time series exhibit long-range correlations and a stationary behavior.

3. Research hypotheses

Hypothesis 1: stock market index has a long-term memory.

Hypothesis 2: exchange rate has a long-term memory.

Hypothesis3: long-term memory of stock market index has a significant relationship with long-term memory of exchange rate.

4. Methodology

This study applied in terms of purpose, regression-based descriptive in terms of method which is based on time series data analysis. Detrended fluctuation analysis is used to examine research hypotheses. Detrended analysis is based on data trend and seasonal changes and parallel market information. Regression equation (1) is applied in detrended fluctuation analysis:

$$\ln\left(F_q(s)\right) = \alpha + h_q \ln s \tag{1}$$

So that, h_q is the Hurst exponent which measures long-term memory of data. Values smaller than 0.5 indicate short-term memory and values greater than 0.5 indicate long-term memory of data. S is the number of observations in N_s sub-observation ($N_s = \left\lfloor \frac{N}{s} \right\rfloor$) of the total detrended N data, and also:

$$F_q(s) = \begin{cases} \left(\frac{1}{2N_s}\sum_{\nu}^{2N_s}[F^2(s,\nu)]^{\frac{q}{2}}\right)^{\frac{1}{q}} & q \neq 0 \\ \exp\left\{\frac{1}{4N_s}\sum_{\nu}^{2N_s}\ln\left(F^2(s,\nu)\right)\right\} & q = 0 \end{cases} \tag{2}$$

In equation (2), $F^2(s,\nu)$ is equal to the mean squares of the second-order polynomial approximation error y_t using the values of time trend data, and we have: $y_t = \sum_{i=1}^{t}(R_i - \bar{R})$

So that y_t represents detrended fluctuations of data. R_i is data fluctuations, which in conventional detrended fluctuation analysis is obtained by differentiating the main values of data from trend values and its seasonal changes. This study applies the ARDL regression model to measure data fluctuations. In this model, the main trend of the data is extracted through the information content contained in its past values and the parallel market information content, and the absolute value of the regression error is considered as random fluctuations of the data. The ARDL models estimated in this study are as described in Equations (3) and (4):

$$\text{Index}_t = \alpha + \sum_{i=1}^{k1}\text{Index}_{t-i} + \sum_{i=0}^{k2}\text{EX}_{t-i} + \varepsilon_t \tag{3}$$

$$\text{EX}_t = \alpha + \sum_{i=1}^{k1}\text{EX}_{t-i} + \sum_{i=0}^{k2}\text{Index}_{t-i} + \varepsilon_t \tag{4}$$

So that, the absolute values of error in model (3), indicate the random fluctuations of the stock index after removing the existing information from the foreign exchange market and the absolute values of the error in model (4), indicate the random fluctuations of the exchange rate after removing the existing information from the stock index. Parameters k_1 and k_2 represent the regression lags, which are estimated based on the minimization of the Akaike criterion in the model fit. It should be noted that in conventional detrended fluctuation analysis, random fluctuations are calculated using Equation (5):

$$r_t = T_t + S_t + R_t \tag{5}$$

Where, T_t represents the value of the main trend of the return data at time t, S_t is the value of the seasonal change component of the data at time t, and R_t is equal to the random fluctuations of the return at time t.

In order to evaluate the long-term memory of exchange rate and stock index, the values of $F_q(s)$ for s=10,11,…,199,200 and for 40 different values of q in the interval (0,4] are calculated and the regression model (1) is fitted separately for each q value. Larger values of q represent more fluctuations in the market, so the hypothesis of long-term memory in the stock market and exchange rate is examined under the assumption of high fluctuations in the market and in different market fluctuations. Also, the value of q = 2 is specifically known as the definite value of the q in detrended fluctuation analysis method.

The data analyzed using R software version 4.0.3 and Eviews software version 10 and at a significance level of 95%. Mean and median, standard deviation, minimum and maximum are used to explain the data.

The statistical population of this research includes Tehran Stock Exchange, in which the information of the total index of the stock exchange is studied. Many studies have confirmed the extensive relationships between the stock market and the foreign exchange market and stock market index and exchange rate in Iran, especially during the last two years, grow simultaneously and in the same direction. Following these facts exchange rate data is studied along with the stock market index. Data with daily frequency during 2014/03/25 to 2021/02/07 is used. The required data has been collected from Exchange Organization site and Gold, Coin and exchange Information Network.

5. Findings

Table 1 provides concentration and dispersion of variables.

According to the results of Table (1), it can be seen that the mean of stock market index and dollar exchange rate during research period is respectively 283673.2 and 827484.86 Rials. The min and max of stock market index are respectively 2065114 and 61163.7. The min and max of dollar exchange rate are equal 318560 Rials and 30300 Rials, respectively. Figure (1) shows the time series values of stock index and exchange rate, as well as the logarithmic returns of these indices during the research period.

Table 1: Descriptive statistics of variables

Variable	Mean	Median	Max	Min	Std.
Index	283673.2	82954.6	2065114	61163.7	442820.1
Exchange Rate	82748.86	39050	318560	30300	67679.32

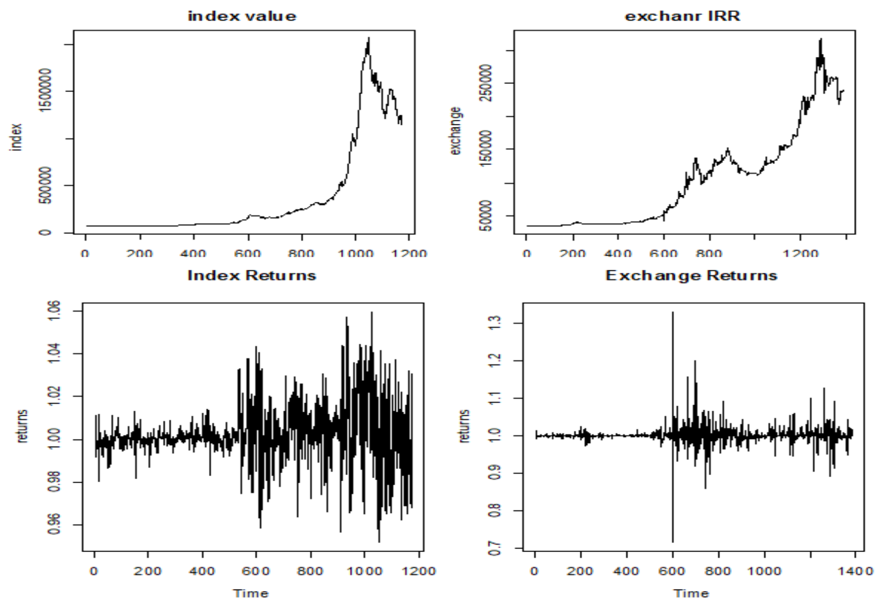

Figure 1: Time series values of index (left) and dollar exchange rate (right) and logarithmic return values

The conventional detrended fluctuation analysis is used to detrend the data. This analysis is performed step by step. Seasonal changes in data are identified and extracted from the data following calculating the data trend based on the 4th order moving average process. The difference between these values and the main data of stock exchange index and exchange rate, calculated by Equation (5), is analyzed as common detrended fluctuations. ARDL model is fitted by HAC method to calculate the random fluctuations under

ARDL models and cross-detrending of data, so that the model results are robust to the assumptions of variance homogeneity and error independence. Table (2) shows the results of fitness of these two variables.

Table 2: Results of ARDL model fitted to estimate random fluctuations of the market

Variable	Explanatory variable	Coeff.	Error	Z-statistic	Sig.	R^2	Model Sig.
Index	$Index_{t-1}$	1.413215	0.082465	17.13718	0.000	0.999434	0.000
	$Index_{t-2}$	-0.499692	0.173626	-2.877973	0.0041		
	$Index_{t-3}$	0.213270	0.110855	1.923865	0.0545		
	$Index_{t-4}$	-0.126774	0.056708	-2.235550	0.0255		
	Ex_t	0.021171	0.271130	0.078084	0.9378		
	Ex_{t-1}	-0.245505	0.085367	-2.875877	0.0053		
	Ex_{t-2}	0.225759	0.156267	1.4447	0.1487		
	C	270.0351	495.3129	0.545181	0.5857		
Exchange Rate	Ex_{t-1}	0.847860	0.052136	16.26236	0.000	0.997864	0.000
	Ex_{t-2}	0.100608	0.049715	2.023701	0.0432		
	Ex_{t-3}	0.046159	0.024865	1.856349	0.0636		
	$Index_t$	0.001825	0.023914	0.076337	0.9392		
	$Index_{t-1}$	0.121087	0.045205	2.678619	0.0059		
	$Index_{t-2}$	-0.027719	0.038345	-0.722892	0.4698		
	$Index_{t-3}$	0.023593	0.035894	0.657291	0.5111		
	$Index_{t-4}$	-0.017726	0.019676	-0.900872	0.3678		
	C	270.0119	150.2140	1.797515	0.0724		

According to Table (2), it is observed stock market index is affected by the values of four previous periods of exchange rate (p-value = 0.0255) and this effect is in the opposite direction. The results of the exchange rate model also show exchange rate is positively affected by the value of stock market index in the past period (p-value = 0.0059). Therefore, based on these findings and the general significance of the models, it can be concluded exchange rate and stock market index incorporate information content relative to each other. Hence, the cross-detrending of parallel markets has been statistically justified. In examining the model assumptions normal distribution of error values is examined using Jarque-Bra test. The significance level of this test is equal to 0.1134 and for the exchange rate model equal to 0.3208, showing the normal distribution of error in these models.

Detrended fluctuation analysis is performed on both detrended fluctuations to estimate the market memory and to measure the relationship between long-term memory of the two markets. After fitting the regression model presented in Equation (1) on two different detrending methods, 40 different values of q and non-overlapping rolling windows s, 40 estimates for the h_q is obtained. Table (3) shows the details of parameter estimation and its significance in these 40 regression models and the stock index.

According to Table (3), stock market index has short memory in conventional fluctuation analysis, while cross-detrending method indicates the presence of long-term memory in stock index data. Therefore, the research hypothesis has not been confirmed under the conventional detrended fluctuation analysis, while the cross-detrending method confirms this hypothesis.

Second hypothesis is examined as well using detrended fluctuation analysis. Table (4) shows the details of parameter estimation and its significance in these 40 regression models for exchange rates.

Table (4) indicates conventional detrended fluctuation analysis is not able to estimate the market memory for q in intervals of [1.4, 2.4]. In other words, if q = 0.1 is the lowest fluctuation in the market and q = 4 is highest fluctuations in the market, the detrended fluctuation analysis method has not been able to estimate long-term memory of market in the conditions of relative and moderate market fluctuations. While according to cross-detrending.

Table 3: Results of estimating h_q of stock index under regression models

q	Common Method		Cross De-trending Method	
	Estimate	Sig.	Estimate	Sig.
0.1	0.231476	0.0000	0.9763	0.0000
0.2	0.216079	0.0000	0.9530	0.0000
0.3	0.21856	0.0000	0.9484	0.0000
0.4	0.22439	0.0000	0.9466	0.0000
0.5	0.230495	0.0000	0.9442	0.0000
0.6	0.235828	0.0000	0.9398	0.0000
0.7	0.239987	0.0000	0.9329	0.0000
0.8	0.242848	0.0000	0.9237	0.0000
0.9	0.244423	0.0000	0.9125	0.0000
1	0.244801	0.0000	0.8994	0.0000
1.1	0.244106	0.0000	0.8859	0.0000
1.2	0.242476	0.0000	0.8715	0.0000
1.3	0.240051	0.0000	0.8568	0.0000
1.4	0.236965	0.0000	0.8421	0.0000
1.5	0.233341	0.0000	0.8276	0.0000
1.6	0.229287	0.0000	0.8134	0.0000
1.7	0.224898	0.0000	0.7997	0.0000
1.8	0.220257	0.0000	0.7865	0.0000
1.9	0.215433	0.0000	0.7738	0.0000
2	0.210483	0.0000	0.7617	0.0000
2.1	0.205456	0.0000	0.7501	0.0000
2.2	0.200391	0.0000	0.7390	0.0000
2.3	0.195321	0.0000	0.7285	0.0000
2.4	0.190271	0.0000	0.7185	0.0000
2.5	0.185264	0.0000	0.7089	0.0000
2.6	0.180315	0.0000	0.6999	0.0000
2.7	0.175438	0.0000	0.6912	0.0000
2.8	0.170643	0.0000	0.6830	0.0000
2.9	0.165939	0.0000	0.6751	0.0000
3	0.161331	0.0000	0.6676	0.0000
3.1	0.156823	0.0001	0.6605	0.0000
3.2	0.152419	0.0001	0.6537	0.0000
3.3	0.148121	0.0002	0.6471	0.0000
3.4	0.143928	0.0003	0.6409	0.0000
3.5	0.139842	0.0005	0.6349	0.0000
3.6	0.135862	0.0007	0.6292	0.0000
3.7	0.131987	0.0011	0.6237	0.0000
3.8	0.128216	0.0015	0.6185	0.0000
3.9	0.124546	0.0021	0.6134	0.0000
4	0.120976	0.0029	0.6086	0.0000

Table 4: Results of h_q of exchange rate under regression models

q	Common Method		Cross De-trending Method	
	Estimate	Sig.	Estimate	Sig.
0.1	0.30647	0.0000	0.9027	0.0000
0.2	0.269009	0.0000	0.8543	0.0000
0.3	0.247748	0.0000	0.8195	0.0000
0.4	0.229648	0.0000	0.7876	0.0000
0.5	0.212318	0.0000	0.7572	0.0000
0.6	0.195109	0.0000	0.7282	0.0000
0.7	0.177844	0.0000	0.7008	0.0000
0.8	0.16051	0.0000	0.6749	0.0000
0.9	0.143157	0.0000	0.6508	0.0000
1	0.12586	0.0004	0.6281	0.0000
1.1	0.108702	0.0024	0.6070	0.0000
1.2	0.091767	0.0116	0.5872	0.0000
1.3	0.075128	0.0420	0.5686	0.0000
1.4	0.058854	0.1170	0.5510	0.0000
1.5	0.043001	0.2598	0.5345	0.0000
1.6	0.027611	0.4764	0.5189	0.0000
1.7	0.012721	0.7467	0.5042	0.0000
1.8	0.001649	0.9671	0.4902	0.0000
1.9	0.015482	0.7027	0.4770	0.0000
2	0.028774	0.4841	0.4645	0.0000
2.1	0.041524	0.3189	0.4525	0.0000
2.2	0.05374	0.2027	0.4413	0.0000
2.3	0.065431	0.1254	0.4305	0.0000
2.4	0.07661	0.0760	0.4203	0.0000
2.5	0.087296	0.0454	0.4106	0.0000
2.6	0.097505	0.0269	0.4014	0.0000
2.7	0.107256	0.0159	0.3925	0.0000
2.8	0.116571	0.0094	0.3842	0.0000
2.9	0.125468	0.0056	0.3761	0.0000
3	0.133968	0.0033	0.3685	0.0000
3.1	0.14209	0.0020	0.3612	0.0000
3.2	0.149855	0.0012	0.3542	0.0000
3.3	0.157279	0.0007	0.3476	0.0000
3.4	0.164382	0.0005	0.3412	0.0000
3.5	0.171181	0.0003	0.3351	0.0000
3.6	0.177691	0.0002	0.3292	0.0000
3.7	0.183929	0.0001	0.3236	0.0000
3.8	0.189908	0.0001	0.3182	0.0000
3.9	0.195644	0.0000	0.3130	0.0000
4	0.201148	0.0000	0.3081	0.0000

method, it is observed that in lower fluctuations (q is in the range [0.1, 1.7]) long-term memory and short memory in highly fluctuated market. In other words, exchange rate memory is short-term in market with high fluctuations, while in markets with small fluctuations, exchange rate has long-term memory. Therefore, according to these findings, the second hypothesis of the research cannot be accepted.

In order to test the third hypothesis of the research, h_q parameters of two variables of stock index and exchange rate have been estimated using both detrending methods. Table (5) shows the results of this test.

According to Table (5), Kolmogorov-Smirnov test confirms the normal distribution of data and so Pearson parametric correlation test is used. The results show the conventional detrending fluctuation analysis does not show a significant relationship between stock index memory and exchange rate memory, but this relationship is confirmed as direct and significant under cross-detrending analysis (r = 0.971). It shows as long-term memory of exchange rate decreases towards short-term memory, the long-term memory of stock index also tends towards short-term memory, and vice versa. Therefore, under this method, the third research hypothesis has been confirmed.

Table 5: The relationship between stock market index memory and exchange rate

<table>
<tr><th rowspan="2">De-trending method</th><th rowspan="2">De-trending Method</th><th colspan="2">KS Test</th><th colspan="2">Correlation Test</th></tr>
<tr><th>z-statistic</th><th>sig.</th><th>Correlation</th><th>Sig.</th></tr>
<tr><td rowspan="2">Common</td><td>Index memory</td><td>0.996</td><td>0.275</td><td rowspan="2">-0.229</td><td rowspan="2">0.156</td></tr>
<tr><td>Exchange rate memory</td><td>0.379</td><td>0.999</td></tr>
<tr><td rowspan="2">Cross De-trending</td><td>Index memory</td><td>0.682</td><td>0.741</td><td rowspan="2">0.971</td><td rowspan="2">0.0000</td></tr>
<tr><td>Exchange rate memory</td><td>0.781</td><td>0.575</td></tr>
</table>

6. Conclusion

The present study investigates the relationship between long-term memory of stock market index and exchange rate using conventional and cross-detrending methods. The results show conventional method finds a short-term memory of stock index, while cross-detrending method shows long-term memory. Therefore, in accordance with these findings, it seems that as previously claimed, detrending methods have significant effects on the results of data long-term memory analysis. Since in multi-fractal nature of financial markets, data trending can be done in different ways, it seems it is possible to develop and improve a standard method of de-trended fluctuation analysis. The results also show exchange rate memory in a market with moderate fluctuations cannot be estimated using the conventional method of detrended fluctuation analysis, while with increasing market fluctuations, short-term memory is appeared. On the other hand, cross-detrending analysis shows with increasing market fluctuations, the long-term memory of exchange rate is reduced and the market has short-term memory. The results also indicate under cross-trending and applying parallel market information in data de-trending, there is a direct and significant relationship between long-term memory of markets. As long as long-term memory in one of markets is reduced and directed towards short memory, the market never behaves as it did. These findings confirm Bahmani Oskooi and Sohrabian (1992). Hosseini Ara (2014), Falahati (2019), Yao et al. (2020) and Manif et al. (2020) confirm the relationship between parallel markets. Based on these findings, it seems that investing in capital markets follows portfolio equilibrium theory. As fluctuation increases in one market, it can be expected that other market fluctuations will increase significantly. The increase in market fluctuations is equivalent to a decrease in long-term memory, which is also discussed multi-fraction theory of financial markets. Therefore, it is proposed to form a portfolio in parallel markets considering increase and decrease in fluctuations and taking into account the similar risks in the financial markets.

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