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Investigating and Comparing Several Centralized Resource Reallocation Methods

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Abstract
Data envelopment analysis is a non-parametric method based on mathematical programming, which is used to evaluate the performance of a set of decision-making units with multiple homogeneous inputs and outputs. One use of data envelopment analysis is the integration of resources and inputs to create new efficient decision-making units. As a classic research topic in the fields of management and economics, the resource allocation problem has gained extensive attention from many researchers, and a large body of research has been conducted accordingly. The present study investigates some of the proposed methods and models of the allocation problem.

Keywords: Data Envelopment Analysis; Centralized Resource Allocation; Most Productive Scale Size; Efficiency

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1. Introduction

Many practical and management problems include a set of comparable and homogenous decision-making units (DMUs) that operate under a centralized decision-maker [1,2]. In such problems, the decision maker (DM) holds sufficient authority and responsibility to control the production process, define the decision-making parameters, manage resource consumption, and perform production planning. In most cases, the available resources are scarce, or it costs heavily to supply them. Therefore, managers constantly seek solutions to gain maximum productivity when consuming these priceless resources. Hence, it is essential to adopt a strategy that, through resource allocation and DMU organization, prevents squandering the resources and inputs that may include costly equipment, maximize productivity, and generate revenue. Many economic entities and enterprises are currently facing issues regarding the reallocation of resources and inputs to gain maximum output and income. For instance, a hospital hires a group of physician specialists for various wards. Needless to say, the board aims to attain specific achievements, such as training specialized workforces, providing high-quality services to clients, generating revenue, and publishing specialized research papers. Clearly, some goals, such as revenue generating and human resource training, are primary objectives, while publishing specialized articles and books is regarded as a middle-level goal. Alternatively, a country's central government may decide to allocate a fixed budget to its provinces to reduce poverty or boost economic and social infrastructures. In such cases, the question that comes up is, considering the tremendous volume of information, what scientific and practical strategies can the centralized DM apply to allocate the fixed budget to the DMUs (here, the provinces) fairly? In the present research, we develop a method that uses the data envelopment analysis (DEA) technique to make all the new DMUs efficient through centralized resource reallocation.

The data envelopment method was first introduced by Charnes, Cooper, and Rhodes [4] and later developed by Banker, Charnes, and Cooper [3], and their proposed models were rapidly and extensively applied for performance evaluation and efficiency measurement of DMUs in various fields of economy, business, and other similar areas. After the early DEA models were developed, researchers decided to suggest methods to improve the efficiency of inefficient DMUs and exploit the potential of inputs. Resource reallocation and DMU reorganization are among these methods applied to make new efficient units. The problem of resource reallocation and DMU reorganization has become one of the most significant practical areas of data envelopment analysis and can provide invaluable insights for resource allocation [5, 6,7,8].

2. Research Literature

Since the resource reallocation problem has become particularly significant in data envelopment analysis, a large and diverse body of research has been conducted regarding this subject. Considering the impact of resource allocation on the size of DMU efficiency, Jolani et al. [9] applied an additive model of DEA for resource allocation. Cooke and Xu [10] adopted an approach that maximizes the efficiency size after allocation using the common set of weights. Amirteimoori & Shafiei et al. [11,12] proposed a method to subtract a fixed value from all DMUs without changing the efficiency size. Hosseinzadeh Lotfi et al. [13] proposed a method based on the enhanced Russell model for centralized resource allocation. Using the inverse data envelopment analysis technique with a cone-ratio structure, Vencheh et al. [14] presented a method for resource reallocation and estimation of DMU's inputs and outputs. Hosseinzadeh Lotfi et al. [15] applied the DEA technique to suggest a method for centralized resource reallocation with probabilistic data. Yu et al. [16] used the DEA technique to investigate human resource allocation in 18 Taiwanese airports. Lozano et al. [17] proposed a method for centralized resource reallocation. Asmild et al. [18] developed a BCC model-based method for centralized resource allocation. Amirteimoori and Kordrostami [19] proposed a method to develop the implementation of supply-and-demand changes in centralized decision-making. Hosseinzadeh Lotfi et al. [20] and Hatami-Marbini et al. [21] introduced the set of common weights approach in DEA and goal programming for resource allocation. Amirteimoori et al. [22] assumed that after the model set and input resources are allocated, all DMUs should be efficient under a set of common weights. Wu et al. [23] proposed a multi-objective linear programming model for resource allocation. Zhang et al. [24] presented a linear programming model with bound variables for resource allocation. Nojoumi et al. [26] conducted a study entitled "Centralized Resource Allocation to Create New Most Productive Scale Size (MPSS) DMUs".

3. A Review of the Centralized Resource Reallocation Research

Lozano & Villa [25] were the first to raise the idea of centralized resource reallocation, and their model was used as the basis of future research on centralized resource reallocation. To reallocate centralized resources, they first integrated inputs and outputs as $\sum_{j=1}^{n} y_{rj}, r=1,2,...,s$, $\sum_{j=1}^{n} x_{ij}, i=1,2,...,m$ and proposed the following radial model:

$$
\begin{aligned}
min \quad & \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) && (1.3)\\
s.t \quad & \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{kj} x_{ij} = \theta \sum_{j=1}^{n} x_{ij} - s_i^-, && i=1,2,...,m\\
& \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{kj} y_{rj} = \sum_{j=1}^{n} y_{rj} + s_r^+, && r=1,2,...,s\\
& \sum_{j=1}^{n} \lambda_{kj} = 1, && k=1,2,...,n\\
& \lambda_{kj} \ge 0, s_i^- \ge 0, s_r^+ \ge 0, \forall k, j, i, r
\end{aligned}
$$

Where s_i^- is the auxiliary variable of integrated inputs, s_r^+ is the auxiliary variable of integrated outputs, and ε is a positive non-Archimedean quantity smaller than any positive real value. Now, let $\left(\theta^*, \lambda^*, \mathbf{s}^{-*}, \mathbf{s}^{+*}\right)$ be the optimal solution for model (1.3). Therefore, the projection of model (1.3), including n new efficient DMUs, takes the following coordinates:

$$\mathbf{x}_k^* = \sum_{j=1}^{n} \lambda_{kj}^* \mathbf{x}_j, \mathbf{y}_k^* = \sum_{j=1}^{n} \lambda_{kj}^* \mathbf{y}_j, k=1,2,...,n$$

As shown in the numerical example of Chapter 4, the model proposed by Lozano & Villa [25] provides no guarantee that new DMUs are MPSS.

3.1 The Proposed Model by Nojoumi, Saati, & Khoshandam

With an aim to create new MPSS DMUs, a method is proposed in [26] for centralized resource reallocation. For this purpose, the following fractional model is solved to gain new MPSS projections:

$$P^* = min \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \qquad (2.3)$$

$$
\begin{aligned}
s.t \quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{kj}x_{ij} \le \theta_i \sum_{j=1}^{n} x_{ij}, \ i=1,2,...,m \\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{kj}y_{rj} \ge \varphi_r \sum_{j=1}^{n} y_{rj}, \ r=1,2,...,s \\
& \sum_{j=1}^{n}\lambda_{kj} = 1, \qquad k=1,2,...,n \\
& \lambda_{kj} \ge 0, \ \theta_i \ge 0, \ \varphi_r \ge 0 \quad \forall k, j, i, r
\end{aligned}
$$

Furthermore, it was proved that the proposed model is always feasible and linear. Later in this work, a dual model was applied to illustrate that all new MPSS DMUs lie on a strong supporting hyperplane.

3.2 The Proposed Model by Meng Zhang et al.

Zhang et al. [27] presented the following model for resource reallocation purposes:

$$
\begin{aligned}
min \quad & \sum_{j=1}^{n}\mathbf{v_j}\left(\mathbf{x}_j + \Delta\mathbf{x}_j\right) \qquad (3.3) \\
s.t \quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{kj}\mathbf{x}_j \le \mathbf{x}_j + \Delta\mathbf{x}_j, \qquad j=1,2,...,n \\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{kj}\mathbf{y}_j \ge \varphi_j^{BCC}\left(\mathbf{y}_j + \Delta\mathbf{y}_j\right), \qquad j=1,2,...,n \\
& \sum_{k=1}^{n}\lambda_{kj} = 1, \qquad j=1,2,...,n \\
& \mathbf{u_j}\left(\mathbf{y}_j + \Delta\mathbf{y}_j\right) = 1, \qquad j=1,2,...,n \\
& \mathbf{v_j}\mathbf{x}_k - \mathbf{u_j}\mathbf{y}_k \ge \mathbf{0} \quad k=1,2,...,n, j=1,2,...,n \\
& \sum_{j=1}^{n}\Delta\mathbf{x}_j \le A \\
& \overline{\mathbf{l}}_\mathbf{j} \le \Delta\mathbf{x}_j \le \overline{\mathbf{u}}_j, \qquad j=1,2,...,n \\
& \lambda_{kj} \ge 0, \mathbf{u}_j \ge \mathbf{0}, \mathbf{v}_j \ge \mathbf{0}, \\
& \mathbf{y}_j + \Delta\mathbf{y}_j \ge \mathbf{0}, \mathbf{y}_j + \Delta\mathbf{y}_j \ne \mathbf{0}, \forall k, j
\end{aligned}
$$

In Model (3.3), $\Delta\mathbf{x}_j, \Delta\mathbf{y}_j, \lambda_{kj}, \mathbf{u_j}, \mathbf{v_j}$ are variables, and φ_j^{BCC} is the efficiency size of DMUs, which is obtained by solving the BCC model in an output-oriented mode. $\overline{\mathrm{l}}_\mathrm{j}$ and $\overline{\mathbf{u}}_\mathbf{j}$ are lower and upper bounds for $\Delta\mathbf{x}_j$ and vector $A=[a_1, a_2, ..., a_m]$ equals a storable total amount of inputs. Model (3.3) is a nonlinear programming problem, and solving this model gives new DMUs, but not all new DMUs are MPSS units, as shown in a numerical example in Chapter 4.

3.3 The Proposed Model by Hosseinzadeh Lotfi et al.

For resource reallocation purposes, Hosseinzadeh Lotfi et al. [28] first proposed the following linear programming problem for DMUs with desirable and undesirable outputs to determine whether the corresponding DMUs are efficient:

$$
\begin{aligned}
max \quad & \varphi_p - \varphi'_p \qquad (4.3) \\
s.t \quad & \sum_{j=1}^{n}\lambda_j\mathbf{x}_j \le \mathbf{x}_p \\
& \sum_{j=1}^{n}\lambda_j\mathbf{y}_j^g \ge \varphi_p\mathbf{y}_p^g \\
& \sum_{j=1}^{n}\lambda_j\mathbf{y}_j^b \le \varphi'_p\mathbf{y}_p^b \\
& \varphi_p \ge 1 \ , \ \varphi'_p \le 1 \\
& \lambda_j \ge 0, \qquad j=1,2,...,n
\end{aligned}
$$

In their proposed model, input $\mathbf{x}_j$ is desirable, but the output vector includes two parts, i.e., desirable and undesirable outputs.

$$\mathbf{y}_j = \left(\mathbf{y}_j^g, \mathbf{y}_j^b\right) \qquad j=1,2,...,n$$

In Model (4.3), maximum desirable outputs and minimum undesirable outputs are assumed, given constraints $\varphi_p \ge 1$ and $\varphi'_p \le 1$ in the optimal solution. Let $\left(\lambda^*, \varphi_p^*, \varphi_p'^*\right)$ be the optimal solution for Model (4.3). Hence, DMUp is efficient if and only if $\varphi_p^* = \varphi_p'^* = 1$ otherwise DMUp is inefficient. Further, they solved Model (4.3) for all DMUs and presented the following resource reallocation model:

$$
\begin{aligned}
& min \quad Z_1 = \mathbf{N} + \sum_{j=1}^{n} \mathbf{v_j} + \sum_{j=1}^{n} \mathbf{w_j} \qquad (5.3) \\
& \max \sum_{j=1}^{n}\sum_{r=1}^{s} \frac{\Delta y_{rj}^{g}}{y_{rj}^{g}} - \sum_{j=1}^{n}\sum_{i=1}^{m} \frac{\Delta x_{ij}}{x_{ij}} \\
& s.t \quad x_{ij} - \Delta x_{ij} \geq \sum_{k=1}^{n} \lambda_{jk} x_{ik}, \qquad j = 1,2,\dots,n, i = 1,2,\dots,m \\
& \quad \varphi_j^{*}\left(y_{rj}^{g} - \Delta y_{rj}^{g}\right) \leq \sum_{k=1}^{n} \lambda_{jk} y_{rk}^{g}, \qquad j = 1,2,\dots,n, r = 1,2,\dots,s \\
& \quad \varphi_j'^{*}\left(y_{tj}^{b} - \Delta y_{tj}^{b}\right) \geq \sum_{k=1}^{n} \lambda_{jk} y_{tk}^{b}, \qquad j = 1,2,\dots,n, t = 1,2,\dots,p \\
& \quad \mathbf{0} \leq \Delta \mathbf{x_j} \leq \mathbf{c_j} - \mathbf{v_j}, \qquad j = 1,2,\dots,n \\
& \quad \mathbf{0} \leq \Delta \mathbf{y_j^b} \leq \mathbf{D_j} - \mathbf{w_j}, \qquad j = 1,2,\dots,n \\
& \quad \sum_{j=1}^{n} \Delta \mathbf{y_j^b} \geq \mathbf{F} - \mathbf{N}, \qquad j = 1,2,\dots,n \\
& \quad \lambda_{kj} \geq 0, \quad j = 1,2,\dots,n, \quad k = 1,2,\dots,p
\end{aligned}
$$

In their proposed model, centralized resource reallocation is conducted, provided that the efficiency size of DMUs remains invariable. They had to solve a linear programming model first and then a multi-objective programming problem.

3.4 The Resource Reallocation Model by Fang Li

The model proposed by Li [29] for resource reallocation based on profit efficiency is as follows:

$$
\begin{aligned}
& max\, \varphi = \sum_{k=1}^{n}\sum_{r=1}^{s} p_{rk}\bar{y}_{rk} + \varepsilon \sum_{i \notin U}\sum_{k=1}^{n} s_{ik}^{-} \qquad (6.3) \\
& s.t \sum_{j=1}^{n} \lambda_{kj} x_{ij} \leq x_{ik}, k = 1,2,\dots,\mathrm{n}, i \in U \\
& \sum_{j=1}^{n} \lambda_{kj} x_{ij} + s_{ik}^{-} = \bar{x}_{ik}, k = 1,2,\dots,\mathrm{n}, i \notin U \\
& \sum_{j=1}^{n} \lambda_{kj} y_{rj} = \bar{y}_{rk}, k = 1,2,\dots,n, r = 1,2,\dots,s \\
& \sum_{k=1}^{n} \bar{x}_{ik} = \sum_{k=1}^{n} x_{ik}, i \notin U, \quad i = 1,2,\dots,\mathrm{m} \\
& \sum_{j=1}^{n} \lambda_{kj} = 1, \qquad k = 1,2,\dots,n \\
& \lambda_{kj} \geq 0, \qquad \forall j,k
\end{aligned}
$$

In Model (6.3), Set U is defined as follows:

= U {i | i^th index, DMUj cannot be integrated}

In addition, $(\bar{x}_{ik}, \bar{y}_{rk})$ are indices of desirable inputs and outputs after centralized resource reallocation. In their proposed model, hence, input indices included two parts:

1. Indices that can be integrated
2. Indices that cannot be integrated

Moreover, p_{rk} is the value of the r^th output component of DMUj with a known quantity, and ε is a positive non-Archimedean quantity. To solve Model (6.3), we first need the values of the output components of DMUs to be available, which is practically a difficult case. For instance, if the number of published papers of each university is considered, it will be difficult to determine its price.

3.5 The Proposed Methods by Jie Wu et al.

Jie Wu et al. [30] presented a study on resource allocation considering environmental factors. In their work, there are n DMUs with coordinates $\left(x_j^1, x_j^2, \boldsymbol{u}_j, \boldsymbol{y}_j\right), j = 1,2,\dots,n$ where x_j^1 is the input vector with components that could be reallocated, x_j^2 is the input vector with components that could not be reallocated, $\boldsymbol{u_j}$ is the undesirable output vector, and $\boldsymbol{y_j}$ is the desirable output vector. In their research, therefore, the input of each DMU is divided into two parts, i.e., $\left(x_j^1, x_j^2\right)$, and the output of each DMU is also divided into two parts, i.e., $\left(\boldsymbol{u_j}, \boldsymbol{y_j}\right)$.

$$x_j^1 = \left(x_{1j}^1,...,x_{pj}^1\right) \geq 0$$
$$x_j^2 = \left(x_{1j}^2,...,x_{pj}^2\right) \geq 0$$
$$\boldsymbol{u_j} = \left(u_{1j},...,u_{oj}\right) \geq 0$$
$$\boldsymbol{y_j} = \left(y_{1j},...,y_{sj}\right) \geq 0$$

For resource reallocation purposes, the following linear programming model is first solved in their proposed method to maximize the desirable outputs:

$$\max \frac{1}{r}\sum_{r=1}^{s} \frac{\sum_{k=1}^{n} \hat{y}_{rk}}{\sum_{j=1}^{n} y_{rj}} \qquad (7.3)$$

$$\begin{aligned}
s.t \quad & \sum_{j=1}^{n} \lambda_{jk} x_{ij}^1 \leq \hat{x}_{ik}^1, && \forall i,k \\
& \sum_{j=1}^{n} \lambda_{jk} x_{ij}^2 \leq x_{ik}^2, && \forall i,k \\
& \sum_{j=1}^{n} \lambda_{jk} y_{rj} \geq \hat{y}_{rk}, && \forall r,k \\
& \sum_{j=1}^{n} \lambda_{jk} u_{rj} = \hat{u}_{rk}, && \forall r,k \\
& \hat{y}_{rk} \geq y_{rk}, && \forall r,k \\
& \hat{u}_{rk} \leq u_{rk}, && \forall r,k \\
& \sum_{k=1}^{n} \hat{x}_{ik}^1 \leq g_i, && \forall i \\
& \lambda_{jk} \geq 0, && \forall j,k
\end{aligned}$$

In Model (7.3), $\hat{u}_k, \hat{y}_k, \hat{x}_k, \lambda_{jk}$ are variables. Further, the following linear programming model is solved in the second stage to minimize the undesirable outputs:

$$\min \frac{1}{o}\sum_{t=1}^{o} \frac{\sum_{k=1}^{n} \hat{u}_{tk}}{\sum_{k=1}^{n} u_{tk}} \qquad (8.3)$$

$$\begin{aligned}
s.t \quad & \sum_{j=1}^{n} \lambda_{jk} x_{ij}^1 \leq \hat{x}_{ik}^1, && \forall i,k \\
& \sum_{j=1}^{n} \lambda_{jk} x_{ij}^2 \leq x_{ik}^2, && \forall i,k \\
& \sum_{j=1}^{n} \lambda_{jk} y_{rj} \geq \hat{y}_{rk}^*, && \forall r,k \\
& \sum_{j=1}^{n} \lambda_{jk} u_{tj} = \hat{u}_{tk}, && \forall t,k \\
& \hat{y}_{rk}^* \geq y_{rk}, && \forall r,k \\
& \hat{u}_{tk} \leq u_{tk}, && \forall t,k \\
& \sum_{k=1}^{n} \hat{x}_{ik}^1 \leq g_i, && \forall i \\
& \lambda_{jk} \geq 0, && \forall j,k
\end{aligned}$$

Where (8.3) is the optimal solution of Model (7.3). In Models (7.3) and (8.3), g_i is the practical upper bound for inputs of x_j^1. As mentioned above, two linear programming models are required to be solved in this research for resource reallocation purposes.

3.6 The Proposed Model by Asmild et al.

Amild et al. [31] presented a research study on centralized resource reallocation based on the BCC model. In their work, they first determine efficient and inefficient DMUs by solving the BCC model for n DMUs with coordinates $\left(\mathbf{x}_j, \mathbf{y}_j\right), j = 1,2,...,n$. The sets of efficient and inefficient DMUs are denoted with E and I, respectively. In the following, they presented the following model for centralized resource reallocation purposes:

$$\min \quad \theta \qquad (9.3)$$

$$s.t \sum_{k\in E}\sum_{j\in I} \lambda_{jk} x_{ik} \le \theta \sum_{j\in I} x_{ij}, \forall i$$

$$\sum_{k\in E}\sum_{j\in I} \lambda_{jk} y_{rk} \ge \sum_{j\in I} y_{rj}, \forall r$$

$$\sum_{k\in E} \lambda_{jk} = 1, \qquad \forall j \in I$$

$$\lambda_{jk} \ge 0, \qquad \forall j, k$$

As can be seen in their model, reallocation is only applied for inefficient DMUs, and efficient DMUs remain unchanged. In their proposed model for inefficient DMUs, an efficient but not necessarily MPSS projection is provided, and efficient DMUs that may not be CCR-efficient remain unaffected. This method demands that the sets of efficient and inefficient DMUs be first determined.

3.7 The Centralized Resource Reallocation Model by Hosseinzadeh Lotfi et al.

For centralized resource reallocation purposes, Hosseinzadeh Lotfi et al. [32] proposed the following linear programming model based on the enhanced Russell model for n DMUs with coordinates $(\mathbf{x}_j, \mathbf{y}_j), j = 1, 2, ..., n$:

$$min \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \qquad (10.3)$$

$$s.t \quad \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{kj} x_{ij} \le \theta_i \sum_{j=1}^{n} x_{ij}, i = 1, 2, ..., m$$

$$\sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{kj} y_{rj} \ge \varphi_r \sum_{j=1}^{n} y_{rj}, r = 1, 2, ..., s$$

$$\sum_{j=1}^{n}\lambda_{kj} = 1, \qquad j = 1, 2, ..., n$$

$$0 \le \theta_i \le 1, \qquad i = 1, 2, ..., m$$

$$\varphi_r \ge 1, \qquad r = 1, 2, ..., s$$

$$\lambda_{kj} > 0, \qquad k, j = 1, 2, ..., n$$

In their non-radial programming model, there are two categories of constraints as $\varphi_r \ge 1, \forall r, \theta_i \le 1, \forall i$, which makes it impossible to have input expansion and output contraction when needed to obtain an MPSS projection for new DMUs, as shown in Numerical Example 2.

4. Numerical Examples

In this section, we discuss two numerical examples to show that the proposed method by Nojoumi et al. [26] outperforms other centralized resource reallocation methods.

4.1 Example 1

Consider ten DMUs in a single input/output case as follows. In the table of Model 4.1, there are ten DMUs named A to J.

Table 4.1: The optimal solutions for CCR and BCC models

DMU	A	B	C	D	E	F	G	H	I	J
x	3	3.5	4	4.5	4.5	5	5.5	6	8	9
y	1	2.5	2.4	2.7	3.6	7.5	4.4	9	7	7
θ_{CCR}	0.333	0.714	0.6	0.6	0.8	1	0.8	1	0.875	0.778
θ_{BCC}	1	1	0.867	0.804	0.924	1	0.844	1	1	0.889

Units A, B, F, H, I, and J are efficient BCC units, and F and H are efficient CCR units. Therefore, unit F and H and all the points between line segment FH are MPSS.

Table 4.2 The projection of DMU with the model of Nojoumi et al [26] and the models of Lozano & Villa and Zhang et al [25].

	Model by Lozano & Villa		Model by Zhang et al		Model by Nojoumi et al	
DMU	x'	y'	x''	y''	x^*	y^*
A	3.5	2.5	5.33	5.33	6	6
B	3.5	2.5	5.25	5.25	6	6
C	3.5	2.5	5.41	3.90	6	6
D	3.5	2.5	5.43	3.52	6	6
E	3.5	2.5	5.32	4.60	6	6
F	3.5	2.5	5.24	5.24	6	6
G	3.5	2.5	5.31	4.25	6	6
H	3.5	2.5	5.22	5.22	6	6
I	3.5	2.5	5.25	5.25	6	6
J	3.5	2.5	5.37	5.37	6	6

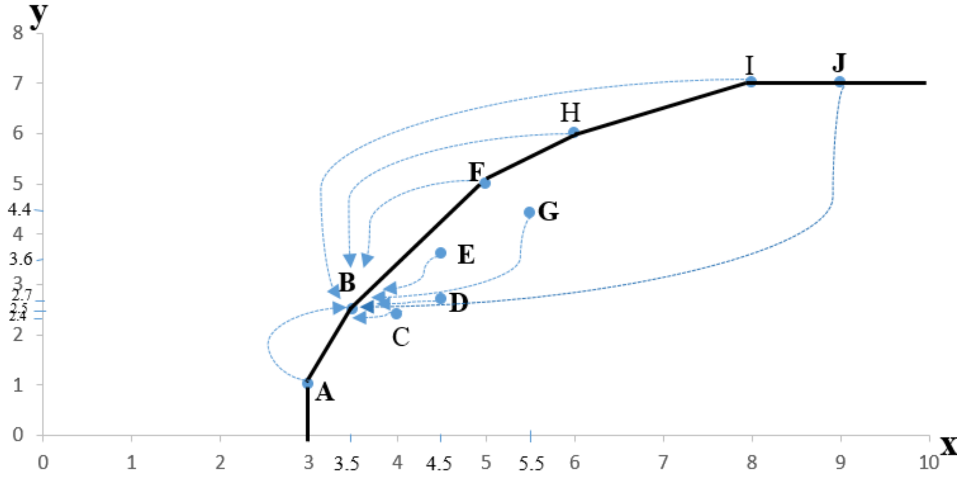

Fig. 4.1: The projection of all DMUs using the model of Lozano & Villa

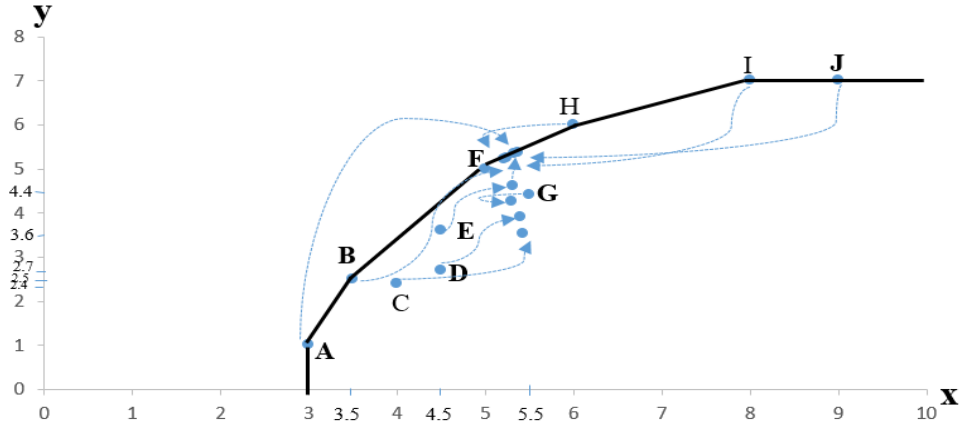

Fig. 4.2: The projection of all DMUs using the model of Zhang et al.

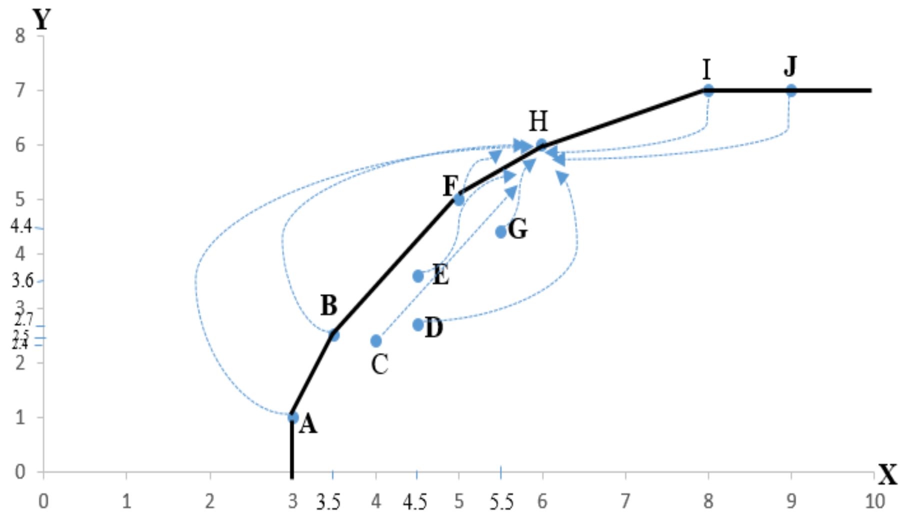

Fig. 4.3: The projection of all DMUs using the model of Nojoumi et al.

As Table 4.2 shows, in the presented method in Lozano & Villa [25], all decision-making units are projected to DMU_B. DMU_B is an efficient BCC (not CCR) unit. The presented model in Lozano & Villa [24] is unable to provide the projection of MPSS for decision-making units. Moreover, efficient units A, B, F, H, and I, along with poorly efficient unit J, are projected on MPSS, but inefficient units C, D, E, and G are not projected on MPSS.

As shown in Fig. 4.2, which depicts the obtained projection from the proposed model in this research, i.e., model (2.3), all DMUs are projected to point H. As shown in Fig. 4.3, DMU_H has the highest constant returns to scale.

4.2 Example 2

As shown in Table 4.3, consider six DMUs in a single input/output case.

Table 4.3: The set of DMUs

DMU	A	B	C	D	E	F
x	6	8	6	8	9	5
y	2	8	4	4	7	2

Here, $\sum_j x_j = 42$ and $\sum_j y_j = 27$ denote the sums of inputs and outputs, respectively. For six DMUs, the optimal solution for the CCR and BCC model is as follows:

Table 4.4 The optimal solution of the CCR and BCC models

DMU	A	B	C	D	E	F
θ_{CCR}	0.33	1	0.66	0.5	0.77	0.4
θ_{BCC}	0.83	1	1	0.75	0.8	1

As shown in the optimal solutions of the CCR and BCC models, DMUb is the efficient unit for the CCR model, and DMUb, DMUc, and DMUf are the efficient units for the BCC model.

Table 4.5 The sets of DMUs for the models proposed by Hosseinzadeh Lotfi et al. [32] and Nojoumi et al. [26]

	The model proposed by hosseinzadeh Lotfi el al.		The proposed model by Nojoumi et al.	
DMU	x'	y'	x''	y''
A	8	8	8	8
B	8	8	8	8
C	8	8	8	8
D	8	8	8	8
E	5	2	8	8
F	5	2	8	8

Table 4.5 details the projection of DMUs using Model (10.3) and the proposed model by Nojoumi et al. [26]. As shown, when Model (10.3) is solved, DMUs A, B, C, and D are projected on an MPSS point, i.e., DMUb, but DMUs E and F are not projected on an MPSS point.

5. Conclusion

The present study reviews and investigates several widely-known proposed methods and models for resource reallocation purposes and compares them with the proposed method by Nojoumi et al. [26]. As discussed in section 3, some of the proposed models are nonlinear, and their methods for centralized resource reallocation include more than a single model. In the work by Nojoumi et al. [26], however, new most productive scale size (MPSS) DMUs are created by solving a single linear model. In production technology, the MPSS points are merited with the quality of exploiting the full potential of inputs for production purposes. In other words, a DMU may be BCC-efficient but with a higher ratio of increasing outputs to increasing inputs or a lower ratio of decreasing outputs to decreasing inputs. Therefore, when the resources and inputs in the production technology include costly equipment, the decision-maker (DM) seeks to find a strategy to prevent squandering these priceless resources, gain maximum productivity through resource consumption management, and exploit the resources' potential to the best.

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