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Research Article

Redundancy Allocation Optimization Based on the Fuzzy Universal Generating Function approach in the Series-Parallel Systems

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Abstract

In This paper we discuss about the redundancy allocation problem when uncertainty exists in the problem. The innovation of this paper is the use of fuzzy universal generating function for calculating the availability of system. The system consists of component in series and for each component, elements are used in parallel. The system has the range of levels for performance from complete perfect to complete failure. Identical redundant elements are included in order to achieve a desirable reliability. The elements of each component are characterized by their cost, weight and availability. These elements are chosen from a list that available in market. To find optimum combination of elements for each component we calculate the system availability and then use proposed NSGA-III. The proposed NSGA-III heuristic determines the maximal availability system configuration base on weight constrain.

Keywords : Reliability, Redundancy allocation problem, series-parallel systems; Fuzzy Universal Generating Function.

1 Introduction

REdundancy allocation is a vital technique used in reliability models to improve the overall reliability of a system. The basic idea of redundancy allocation is to add extra components or resources to a system, which can increase its ability to tolerate failures or malfunctions. Redundancy allocation involves determining the optimal number, placement, and type of redundant components to be added to the system to achieve a desired level of reliability at a minimum cost. This technique has been extensively studied in the field of reliability engineering, with numerous approaches and models proposed to address different types of redundancy allocation problems. The selection of an appropriate redundancy allocation strategy can play a crucial role in ensuring the reliability and safety of critical systems, such as aircraft, nuclear power plants, and medical devices. In traditional studies the reliability of elements has been considered binary, both systems and elements can only take two possible states: completely work and totally failed [1]. In this research we assumed that the system has the range of levels for performance from complete perfect to complete failure. Such redundancy allocation problem is called multi sate system (MSS). In description of this MSS can noted to one example: consider a pomp that his work is providing a sufficient flow of water for cooling system. This pomp has a range of performance

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that in each state can provide a specific flow that this flow is differ in different situation. An example of this subject is that the pomp is working in what hours in day has been affected in pomp performance.

The redundancy allocation problem (RAP) is a complex combinational optimization problem, which has very importance in many industrial applications, for example in electronic systems, power stations and manufacturing production systems [2]. As an example, a power generation station has multiple power generation units connected in parallel. Each unit might have multiple state, e.g., 50MW (full capacity), 30 MW and 0 MW (failed state) [2]. Reliability analysis considering multiple possible states is known as multi-state reliability analysis [3, 4, 5, 6]. Many binary system availability structures have been extended to multi state systems [1, 7, 8, 9]. In this paper we focus on series-parallel systems. In a series-parallel structure, the systems consist of components in series and for each component multiple element choices are used in parallel. The configuration of such system is depicted in Fig 1.

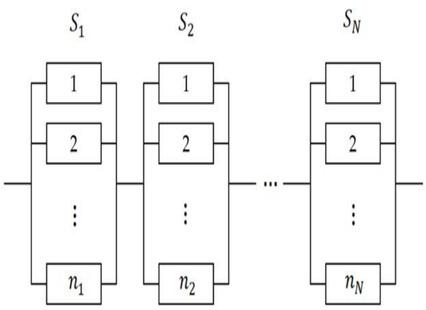

Figure 1: A series-parallel system

The goal of design is to determine the optimal combination of elements to maximize availability of system and minimize cost while meet weight constrains. The RAP is NP-hard [10]. In most studies in case of RAP consider the system in binary state. The RAP for binarystate series-parallel systems has been studied in many different forms, and by considering numerous approaches and techniques, as summarized in [11], [12] and [13]. But in this paper we consider the system in multi-state. In this paper, a more practical approach is proposed for the redundancy allocation Problem optimization of series-parallel systems. Two factors are identified that might affect system availability: (1) version of the selected elements (2) redundancy i.e., number of elements in various components. M. Shahriari proposed a solution for the redundancy allocation problem (RAP) for a system with repairable components and k-out-of-n sub-systems. The objective is to maximize system reliability while considering cost and weight constraints. The study uses a genetic algorithm (GA) for solving the Np-hard problem, with response surface methodology (RSM) for parameter tuning, and a simulation method for reliability calculations. A numerical example is presented to demonstrate the algorithm's performance. [36] There are some exact algorithms in literature for solving RAP. For example, Kuo et al [14] use Dynamic programming (DP) to solve this problem. Also, from Branch and Bond algorithm (B&B) used by Kuo et al [14] and Ha and Kuo [15]. But, the literature of meth-heuristic is very rich including Tabu search [16], Genetic algorithm [17], [18] and ant colony optimization [19]. We use fuzzy universal generating function method and NSGA-III algorithm to solve this problem. The FUGF technique started by Yi Ding in 2008 [20]. And The NSGA-III algorithm is used in many fields, for solving the multi-objective problems. Table 1 provides a summary of the recently adapted meta-heuristics for solving the RAP.
Shahriari M. presented a solution for the redundancy allocation problem in a series-parallel system, aiming to maximize system reliability while minimizing costs. The problem is solved using various algorithms, including Non-Dominated Sorting Genetic Algorithm II, and considering weight and volume constraints. The system has k-out-of-n subsystems and time-dependent component failure rates, with the redundancy policy affecting the working component failure rate. [37]

Summarized result of the study of the shows that:
1- The Redundancy Allocation Problem (RAP) is a type of reliability optimization problem, and the RAP for Multi-State Systems (MSSs) is particularly difficult. The Multi-State Series-Parallel System (MSSPS) is a commonly used structure in MSSs, but there is no exact approach to solve MSSPS RAP in the literature. The authors propose a new method based on dynamic programming to solve this problem. There are two for-

Table 1: Results.

Authors	Year	Objective No.	Main adapted meta-heuristic (Outstanding)	Solution method
Sharifi et al. [37]	2019	multi	MODM	GA
Zaretalab et al. [21]	2020	Single	MA	GA
Wang et al. [22]	2020	Multi	NSGA-II	—
Sharifi et al. [40]	2020	Single	GA	—
Guilani et al. [23]	2020	Single	GA	—
Sharifi Taghipour [24]	2021	Single	GA	—
Hsieh [25]	2021	Single	SSO	PSSO, ICA,
			NAFSA, SFS, PGA	
Zhang et al. [26]	2021	Single	IABC	IA, PSFS, NAFSA, ABC
Fu Li [42]	2022	multi	Stochastic Optimization	—
Shuai Li et al. [41]	2022	multi	particle swarm optimization	—
Thymianis et al. [43]	2023	multi	Bat and Firefly Algorithm	—

mulations of RAP, which are maximizing system reliability and minimizing system cost under resource constraints. The authors investigate the conversion between these two formulations to solve both via their proposed method. The results of experiments conducted on benchmark systems show that the proposed method outperforms existing meta-heuristic methods and finds new global-optimal solutions. The proposed method is particularly effective on systems with more subsystems and fewer component types, providing exact optimal solutions in less running time.

2- Redundancy allocation problem (RAP) for multi-state series-parallel systems (MSSPS) and continuous-state series-parallel systems (CSSPS) with uncertain component parameters, such as costs and reliabilities. Existing research mainly focuses on binary-state RAP with data uncertainties, with few studies on epistemic uncertainty in MSSPS RAP and none in CSSPS. The authors propose a common model for both systems, handling the uncertainty through state distribution and interval values for costs. A robust model is established to deal with uncertain parameters and is reformulated as a mixed-integer linear programming problem via duality theory. Numerical experiments demonstrate the robustness of the proposed model under different conservative levels, the effectiveness of robust solutions against uncertainty, and a comparison with stochastic programming models.

3- The reliability-redundancy allocation problem (RRAP) mainly focuses on source-to-sink systems. This paper proposes a new formulation called the g-reliability RRAP (gRRAP), which considers imperfect nodes and regards nodes and links of the system as subsystems. g-reliability is the probability that all nodes of the system are connected through normal links, which is suitable for complex modern network systems. The paper proposes an algorithm to calculate the objective function of the gRRAP and develops an improved particle swarm optimization algorithm with a specific particles initialization approach (PIPSO) to solve the gRRAP. The experimental results show that the PIPSO algorithm outperforms several popular metaheuristic algorithms and previous works in solving the gRRAP and four benchmark problems. The paper also proposes a measure to evaluate the uniformity of particle position, showing that particle swarm optimization with more uniform initial particle distribution converges faster and better.

4- Examines the effectiveness of hybrid schemes that combine individual nature-inspired algorithms in solving reliability and redundancy allocation problems with multiple goals. The study implements known algorithms from literature and tests nine hybrid schemes on ten well-known case studies in reliability engineering. The results indicate that hybrid schemes outperform individual algorithms, supporting the hypothesis that hybridization can improve optimization methods. The best-performing hybrid scheme in the study is the combination of Bat Algorithm and Firefly Algorithm.

2 Problem Formulation

In this section we present assumptions, notations and model for this problem.

2.1 Assumptions

1) Weight and cost for each element are known and deterministic.
2) There are no supply constraints for elements.
3) Failed elements are not repaired.
4) The number of components is fixed.
5) The elements are independent.
6) An element has different performance rate in different states
7) The performance rate of a multi-state series-parallel system equal the performance rates of its component.
8) State probabilities of an element have a fuzzy value.

For more, the elements are chosen from the list that existed in market but when one type chosen for a comment, other chooses should be from same type. All redundancy strotedies are active. Switching to element is done perfectly.

2.2 Notation

S: Number of sub-systems
i: index for component, $i \in \{1, 2, , s\}$
j: index for element version in a component, $j \in \{1, 2, , n_i\}$
$C(X)$: total cost of designed system
W: weight constraint
$A(\omega)$: availability of designed system
n_i: number of element type that available for i^{th} component
c_{ij}: cost of j^{th} element available for i^{th} component
w_{ij}: weight of j^{th} element available for i^{th} component
x_{ij}: quantity of j^{th} element used for i^{th} component in the parallel state
L_i: minimum number of elements that can be used in the parallel state for i^{th} component
U_i: maximum number of elements that can be used in the parallel state for i^{th} component
ω : minimum demand availability for designed system

2.3 The optimization model

Based on the discussion above, the optimization model for series-parallel system can be expressed as follows:

$$\begin{aligned}
&Min \quad C(X) = \sum_{i=1}^{n}\sum_{j=1}^{n_i} C_{ij}x_{ij} \\
&Max \quad A(\omega) \\
&s.t. \\
&\qquad \sum_{i=1}^{n}\sum_{j=1}^{n_i} w_{ij}x_{ij} \leq W \qquad i = 1, ..., s \\
&\qquad\qquad\qquad\qquad\qquad\qquad j = 1, ..., n_i \\
&\qquad L_i \leq \sum_{j=1}^{n_i} x_{ij} \leq U_i \qquad i = 1, ..., s
\end{aligned} \tag{2.1}$$

Constraint (1) express that total weight of designed series-parallel system should be less than W . constraint (2) expresses the minimum and maximum redundancies for each component.

3 The proposed FUGF method and NSGA-III

In this section we describe the method that used for solving the RAP in series-parallel system while the elements have a range of performance. The innovation of this research is that the states probabilities are fuzzy value and this subject in combination to MSS, our work in solving the problem makes hard.

There are many methods that used for solving the redundancy allocation problem. For example, Noura Beji used SWARM algorithm with Local search [26] to solving this problem or Nabli Nahas used Ant colony algorithm [27] or Didier used genetic algorithm and simulation [28] or Ouzineb used Tabu search [29] to solving RAP.

In this paper we firstly use fuzzy universal generating function to calculate the availability of system and then propose NSGA-III method has been used to find the best solution.

3.1 The fuzzy universal generating function

Since the number of MSS states increases very rapidly with the increase in number of

its elements, the universal generating function (UGF) was firstly proposed in [27] to reduce computational complexity for the MSS. The further developments and applications of UGF technique were presented in [28, 29, 30] and detailed description can be found in [31] that summarized recent achievements in the field [20]. the logic of universal generating function has been described with ushakov in 1986 that detailed description exist in [27]. But in this research, we used Fuzzy universal generating function to calculating the availability of system. M. Shahriari proposed an extension of the VIKOR method for multiple criteria decision-making problems using Atanassov intuitionistic fuzzy sets. The proposed intuitionistic fuzzy VIKOR measures the satisfaction and dissatisfaction degree of each alternative with respect to each criterion, using degrees of membership and non-membership. The method uses linguistic variables expressed in intuitionistic fuzzy numbers and IFS aggregation operators to identify the most suitable alternative among potential alternatives based on subjective judgment and objective information. Practical examples demonstrate the proposed method's procedure. [38]
As discussed in above, two basic assumptions of the conventional MSS model cannot be satisfied by some MSSs. Therefore, conventional UGF techniques cannot be directly used to analyze these systems. Integration of the MSS model and fuzzy set techniques can be in keep with the need of reliability assessment for such systems.
The suggested FMSS model are based on the following two assumptions corresponding to the assumptions for the conventional MSS, respectively:

1) state probabilities of a component can be measured as fuzzy values; and
2) state performances of a component can be accounted by a fuzzy way.
In this paper we assumed that state probabilities have fuzzy value. For calculating the availability to such RAP with fuzzy state probabilities, we use Fuzzy universal generating function (FUGF). the logic of this technique and practical example exists in [20].
Yi Ding in 2008 [20] describe the UGF and FUGF technique to calculating the series-parallel multi state availability. In this paper we used from result of this research that done with YI Ding.

4 Numerical example

In this section we use a numerical example to demonstrate the efficiency of our method that used to solving this problem. The problem that discussed in this paper is the redundancy allocation problem for multi-state series-parallel systems that within the state probabilities have fuzzy value. There is a system consist of components (subsystems) in series that in each component the elements are in parallel. With regarding the assumptions that described above, a numerical example provided.
In this example we consider a system with four components (subsystem) that for each subsystem four type elements exist that the elements characterized by this cost, weight and performance rate. The goal of problem is finding the best type element for each component and determines the number of elements in each component.
Specification of elements exists in table 2. This table shows the cost, weight and performance rate for each element that can be chosen for components. The maximum weight of system is 4. And the minimum and maximum number of elements that can be allocated is one and three. The Popsize in this example is 30 and number of generations is 50.

This problem coded in MATLAB and runes with a personal computer with Intel Pentium 4 and 2.8 GHZ CPU. The time that lifted to running this problem was 68 minutes. The set of solutions that are MATLAB outputs shown in table 3 and figure 2. Because of this model is two objectives, there are no one solution and there are a set of solution.

The Pareto diagram for this set of solution shows in figure 3. In this figure availability and cost of system for each allocation has been shown.

The Pareto diagram is a graphical representation of the trade-off between two or more objectives. In the context of the Redundancy Allocation Problem (RAP), a Pareto diagram can be

Table 2: Results.

sub system	Element type	performance rate				cost	weight
		0	0.5	0.8	1		
1	1	(.0225,.03,.0335)	(.2,.24,.28)	(.231,.236,.24)	(.24,.29,.32)	2.545	0.545
	2	(.145,.165,.170)	(.224,.232,.24)	(.125,.150,.185)	(.225,.235,.245)	1.826	0.826
	3	(.150,.155,.165)	(.02,.025,.035)	(.10,.15,.17)	(.22,.26,.28)	3.9	0.975
	4	(.015,.035,.065)	(.24,.27,.29)	(.12,.17,.21)	(.038,.041,.052)	4.08	1.08
2	1	(.05,.08,.12)	(.045,.050,.055)	(.010,.034,.039)	(.284,.291,.297)	3.25	0.55
	2	(.04,.043,.048)	(.14,.15,.16)	(.23,.237,.242)	(.26,.30,.34)	3.13	0.63
	3	(.08,.12,.15)	(.15,.19,.21)	(.22,.28,.30)	(.114,.122,.126)	2.4	0.74
	4	(.03,.09,.14)	(.024,.032,.042)	(.261,.267,.272)	(.23,.28,.34)	1.12	0.9
3	1	(.145,.155,.165)	(.125,.185,.205)	(.130,.170,.220)	(.10,.20,.35)	4.15	1.15
	2	(.110,.125,.130)	(.,1400,.1405,.1505)	(.125,.136,.143)	(.038,.043,.050)	2.25	0.25
	3	(.08,.11,.18)	(.065,.075,.090)	(.123,.132,.140)	(.032,.045,.055)	1.08	0.38
	4	(.05,.08,.1)	(.035,.065,.095)	(.137,.142,.146)	(.124,.131,.136)	3.49	0.494
4	1	(.074.,084.094)	(.155,.158,.161)	(.186,.192,.203)	(.190,.240,.290)	2.25	0.625
	2	(.035,.045,.055)	(.033,.040,.047)	(.115,.155,.185)	(.047,.052,.058)	4.14	0.79
	3	(.029,.031,.034)	(.17,.25,.31)	(.14,.18,.20)	(.05,.10,.15)	3.15	0.875
	4	(.11,.22,.33)	(.04,.042,.044)	(.195,.205,.215)	(.131,.133,.134)	2.08	0.545

Table 3: the elements that allocated to components with availability and cost of system for each allocation

allocation	availability	cost
a	0.533	8.286
b	0.5243	7.436
c	0.8886	17.82
d	0.7617	12.075
e	0.5681	10.175
f	0.6499	11.345
g	0.7239	11.356
h	0.7831	17.65
i	0.5105	6.276
j	0.6274	10.626
k	0.5519	9.005

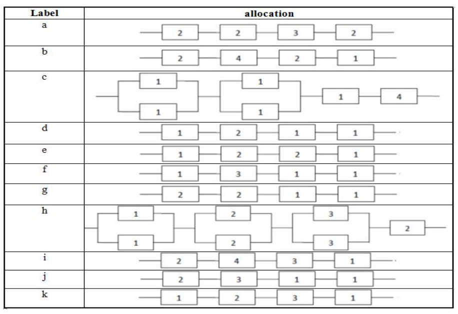

Figure 2: Type of allocation

used to depict the relationship between the availability and cost of the system for each allocation in figure2. The Pareto diagram shows the feasible solutions that achieve the optimal trade-off between availability and cost.

The Pareto diagram typically consists of a scatter plot, with availability on the y-axis and cost on the x-axis. Each point on the plot represents a different allocation of redundancy to the system components. The optimal solutions, which lie on the Pareto front, are those that cannot be improved in one objective without sacrificing the other. The Pareto front represents the set of non-dominated solutions, where no other solution exists that is better in both objectives.

The Pareto diagram provides valuable insights for decision-makers when selecting the optimal design solution. By analyzing the Pareto front, decision-makers can identify the trade-offs between the objectives and select the optimal solution based on their priorities. For instance, if the decision-maker prioritizes availability over cost, they would select the solution with the highest availability, even if it has a higher cost.

In summary, the Pareto diagram is a powerful tool for visualizing the trade-off between objectives in the RAP. It provides valuable insights for decision-makers by identifying the optimal design solutions that achieve the best trade-off between availability and cost.

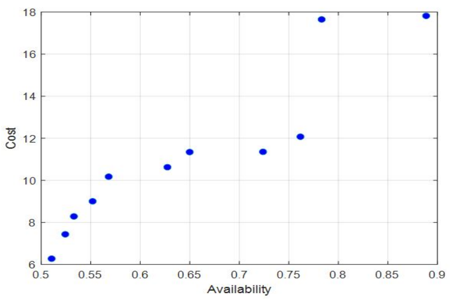

Figure 3: Pareto diagram

5 Conclusion

This research is focused on Redundancy Allocation Problem (RAP) with a number of components that corporate in series state and a number of elements for each component that can be corporate in parallel state.
In this work, we develop an approach for solving the Redundancy allocation problem for multi-state series-parallel systems with fuzzy state probabilities. In this approach we used Fuzzy Universal Generating Function (FUGF) to calculate availability of system and a NSGA-III method has been used to finding the optimum design.
Considering fuzzy value for state probability is the innovation of this research. And a mathematical method used for calculating availability of such system. This method is fuzzy universal generating function.
In this study, proposed an innovative approach for addressing the Redundancy Allocation Problem (RAP) in multi-state series-parallel systems with uncertain state probabilities. We introduce the use of Fuzzy Universal Generating Function (FUGF) to calculate the system's availability, which considers the uncertainties associated with state probabilities. Furthermore, we adopt a NSGA-III algorithm to identify the optimal design solution for the problem. This approach provides a more robust and accurate solution to the RAP compared to traditional methods, which may not account for uncertainties in the system state probabilities.
The primary contribution of this study is the consideration of fuzzy values for the state probabilities of the system. This approach is essential as it allows for more realistic modeling of complex systems, where uncertainties are inherent. Additionally, the use of FUGF for calculating the availability of the system provides an efficient and effective means for handling fuzzy probabilities. This method has proven to be highly accurate and robust, and it can be used to analyze the reliability of a wide range of systems.
To summarize, the proposed approach for addressing the Redundancy Allocation Problem (RAP) in multi-state series-parallel systems with fuzzy state probabilities is a significant and valuable contribution to the field of reliability engineering. The utilization of Fuzzy Universal Generating Function (FUGF) and NSGA-III algorithm presents an innovative and powerful approach for designing reliable and safe systems.

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