A Hybrid Model for Portfolio Optimization Based on Stock Price Forecasting with LSTM Recurrent Neural Network using Multi-Criteria Decision Making Techniques and Mean-CvaR

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ABSTRACT

The importance of a price forecasting issue due to the market volatility is very substantial. Investors have no desire to tolerate the high risk and invest in such a market due to avoiding ambiguity and mainly looking for a suitable solution for investing with high returns and low risk. The purpose of the research is to combine decision-making techniques with recurrent neural networks to create and develop a mathematical model for stock portfolio optimization due to different time horizons. Therefore, the top ten industries were selected using the Fuzzy Analytical Hierarchy Process (FAHP), according to effective criteria on the active industries in the stock market and using the opinions of active industries' experts, between May 2016 and May 2021. Then the price of stocks was forecasted in intended time periods using Long Short Term Memory RNN. In the next step, three stock portfolios with the short-term, mid-term, and long-term time horizons were created using a Combined Compromise Solution method, and then the optimized weights of each stock in the different portfolios were defined, and an efficient frontier was drawn by using Conditional Value at Risk (CVaR). The results showed that the provided model has high efficiency in stock portfolio optimization.

Keywords: Optimization, Fuzzy Analytical Hierarchy Process (FAHP), Combined Compromise Solution (CoCoSo), Long Short Term Memory (LSTM), Conditional Value at Risk (CVaR)

1. Introduction

The risk and return are two important factors that investors should consider in financial market activities. For the first time, Markowitz proposed the classical model of portfolio optimization (mean-variance model), which is the basis of modern portfolio theory, for portfolio management is based on two factors, risk, and return (Markowitz, 1952). In the classic Markowitz model, the variance was considered a measure of portfolio risk, but studies have shown that variance cannot be an accurate measure of risk (Lwin et al., 2017). The Value at Risk (VaR) was introduced as a suitable measure to measure the risk of the investment portfolio after the expansion of studies (Lwin et al., 2017). VaR is hampered by optimization performance due to some limitations such as it is non-convex and non-smooth and has multiple local minimum, which makes the problem difficult to seek the global minimum. Additionally, this measure is not continuous for discrete distributions. Therefore, Conditional Value at Risk (CVaR) was developed by Rockefeller and Uryasev to measure risk (Uryasev, 2000). In order to make an optimal portfolio, forecasting the price of assets used in the investment portfolio is also an important issue that many researchers have addressed (Ghaffari-Nasab et al., 2011). The success factor of making an optimal investment portfolio is accurate risk estimation and a true prediction of the return on assets. In recent years, advances in artificial intelligence have led to innovations in the financial industry. Among them, we can mention the selection of investment portfolios based on mathematical models and forecasting techniques (Gârleanu & Pedersen, 2013). Since portfolio models are large, non-convex, nonlinear, and discrete and fall into the category of NP-Hard problems (LI & XIAO, 2013), the use of traditional algorithms to optimize despite their simplicity is less accurate (Haddad, 2019). Therefore, in order to increase the accuracy of optimization and selection of the most efficient stock portfolio in this study, we intend to optimize portfolios created in three- time horizons (short, mid and, long term) by combining Fuzzy Analytical Hierarchy Process (FAHP), combined compromise solution (CoCoSo), Long Short Term recurrent neural network (LSTM) and CVaR.

Theoretical principles and review of research background

One of the major concerns of Investors in the field of financial decision-making is to choose the portfolio that provides the highest return with the least acceptable risk, within a certain time horizon. In cases in which there are a lot of data or options for decision making, adopting an appropriate approach and strategy to use the overall capacity of the market and increase the wealth of investors is a goal that researchers seek to achieve. Therefore, it seems necessary to provide a mathematical model that can choose the most appropriate option among the various options. FAHP and CoCoSo techniques are a combination of Multi-Criteria Decision-Making (MCDM) methods that can be used in such issues. The process of FAHP, when priorities indicate uncertainty and inaccuracy, was proposed to deal with ambiguity by using fuzzy numbers and calculations and combining them with the Analytical Hierarchy Process (AHP) to solve decision problems. Chang introduced the development analysis model, which has been used more than any other method for FAHP in 1996 (Chang, 1996). Chang used the concept of the feasibility degree or probability of being bigger degree in order to generalize the technique of the AHP to fuzzy space. The degree of feasibility is to determine how likely a fuzzy number is to be greater than another fuzzy number. Numerous studies have used FAHP for ranking and selection. Amir Hosseini and Ghobadi evaluated and selected the stock portfolio using fuzzy theory and Multi-Criteria Decision Making. The results showed a 72% rate of return on the selected portfolio(AmirHossini, 2016). GhaffariNasab et al. used two techniques of FAHP to select a stock portfolio of pharmaceutical companies operating on the Tehran Stock Exchange (Ghaffari-Nasab et al., 2011). Lashgari and Safari used FAHP to select the stock portfolio. The results showed the efficiency of the proposed model (Lashgari & Safari, 2012). In CoCoSo, a combined compromise solution is presented using a combination of simple additive weighting (SAW) and weighted product model (WPM) to rank the options. The ultimate goal of this ranking method is to select a number of options based on a number of criteria. This technique was first proposed by. (Yazdani et al., 2019). Peng et al. used a combination of CoCoSo techniques with fuzzy information to create a decision-making model in the

Chinese stock market. The used technique performed better in selecting and identifying the Chinese stock market bubble than traditional decision-making methods (Peng & Luo, 2021). By combining CoCoSo with AHP, Sivam and, Rajendran optimally optimized multi-objective production planning. Their research showed that the created model has a better performance compared to other methods (Sivam & Rajendran, 2020). Classic portfolio optimization models often use historical data to calculate the expected return. This is because using historical returns, at least in the short-term horizons of the future, gives inaccurate estimates (Freitas et al., 2009), and also because stock prices in the short term are heavily influenced by investor behavior, it does not make sense to use the average historical returns to obtain the expected returns. Therefore, in order to create a favorable investment, combining stock return forecasting with the portfolio optimization model can improve the performance of the created portfolio and give more returns to the investor. Various methods have been used in various studies for forecasting the return on assets. Including Recurrent Neural Networks (RNNs), which are a family of Neural Networks that are specifically designed to process and forecast the movement of time series or three-dimensional data (Haykin & Network, 2004). These networks are inspired by human learning in their operating system. In these types of networks, there are loops to enable the repetition and remembrance of information. Simple Recurrent Neural Networks are not useful for financial time series data due to their limited data remembrance. To investigate the long-term dependence of data, the LSTM was introduced in 1997 by Hockeriter and Schmidber (Patterson & Gibson, 2017). These networks are among the strongest Recurrent Neural Network architectures that can be taught long-term dependencies and the system is able to learn the training received. The Studies showed that the LSTM performs well in forecasting time series data. Dezsei and Nistor forecasted the future stock price of the Romanian stock market by using LSTM. Comparing classical forecasting tools with LSTM, they found that the LSTM model performed better and reduced risk (Dezsi & Nistor, 2016). Samroichrama and Fernando compared the accuracy of the Recurrent Neural Network, a simple Recurrent Neural Network, LSTM, and Gated-Recurrent Unit (GRU) methods in predicting the futures prices of listed companies in the Colombo Stock Market. They have fewer errors than other methods. They also stated in their research that the GRU model has the highest error rate (Samarawickrama & Fernando, 2017). Chen et al. predicted the price in an article entitled Using LSTM to forecast Stocks price in the Chinese Stock Market. They used a thirty-day historical stock price sequence with ten learning features to train the system. The results showed that the LSTM model performed better than the prediction method (Chen et al., 2015). In the field of portfolio risk measurement and optimization, Nasini et al. used CVaR to measure the portfolio of shares of American Companies (Nasini et al., 2021). Haddadi et al. optimized the portfolio using MAD and CVaR criteria and compared it with classical and meta-innovative methods. The results show that the NSGA2 meta-innovative method showed more risk in MAD and CVaR criteria compared to the classical method in solving the portfolio optimization problem (Haddadi et al., 2021). Rarei et al. In a study of the Mean-CVaR model and the symmetric and asymmetric conditional heterogeneity approach optimized the stock portfolio of 30 listed companies. The results showed that the use of CVaR model instead of traditional risk models is significantly effective in improving performance (Raei et al., 2020). Karimi and Goodarzi-Dehrizi optimized the stock portfolio using the imperialist competitive algorithm and the particle swarm algorithm under the CVaR criteria. The results showed that the risk and return of the two algorithms are not statistically significant, but the imperialist competitive algorithm achieves the optimal result in a shorter time (Karimi & goodarzi dahrizi, 2020). RahnamaRoodposhti et al. solved the problem of optimizing the stock portfolio of private companies by using the bee colony algorithm in the case of data shortage. The results showed that an optimal portfolio is a portfolio that has a combination of low-risk and high-risk assets (Rahnama Roodposhti et al., 2018). Navidi et al. performed a three-step approach to stock portfolio optimization for each company using CVaR measure at three confidence levels of 95%, 99%, and 90%. By comparing the obtained results, they found that the results obtained from the higher confidence level are more accurate than the answers obtained from the lower confidence level (Navidi et al., 2016). RahnamaRoodposhti et al. examined the performance of portfolio optimization based on a stable model with classical optimization in predicting portfolio risk and return. They showed that the predicted return of the portfolio in the stable model is not significantly different from the predicted return in the classical model and the risk predicted in the stable model is not significantly different from the risk predicted in the classical model. However, by examining the return and risk of portfolios formed based on the weight presented by each of the models, it was found that in the Iranian market, the real returns of both methods are not significantly different from each (Rahnam Roodposhti et al., 2015). Researches are summarized in table 1.

A review of the literature of studies related to the field of this study showed that few studies have optimized the portfolio based on stock price forecast (Chen et al., 2021; Fekri & Barazandeh, 2019; Ma et al., 2021) and there are shortcomings in this regard. Among them, the following can be mentioned:

Select a stock based only on the price forecast factor. Failure to simultaneously consider the technical and fundamental criteria for decision-making and selecting the stock. Using inefficient (traditional retrospective) methods for forecasting stock prices.

In addition, Investment-fund managers, despite spending huge amounts of money to hire and employ financial analysts to select the best portfolio, their portfolio of choice has not achieved the expected return from the market and this has led to dissatisfaction among investors. Also, Individual-investors are not able to make the good decision in the use of their personal financial resources due to lack of financial knowledge and a lack of a systematic method, so according to the above and since the optimization problem is in the category of NP-Hard problems (Nasini et al., 2021), In this research, we will try to introduce a mathematical model for portfolio optimization from a combination of three methods of stock price forecasting (using LSTM recursive neural network), Multi-Criteria Decision Making and Mean-CVaR model, as well as considering technical and fundamental criteria.

Table 1 papers on portfolio optimization

The studies	Optimization Technique	Forecasting Techniques	Decision Making Techniques
(Peng & Luo, 2021)	-	-	CoCoSo
(Sivam & Rajendran, 2020)	-	-	CoCoSo,AHP
(Lee & Yoo, 2020)	-	RNN, LSTM, GRU	-
(Dezsi & Nistor, 2016)	-	LSTM	-
(Raei et al., 2020)	-	RNN, LSTM	-
(Fekri & Barazandeh, 2019)	Markowitz Mean-Variance-Skewness	RNN	-
(Centeno et al., 2019)		ARIMA	-
(Ma et al., 2021)	Omega Model و Mean-CVar	LSTM, DMLP, Convolutional Neural Network	-
(Navidi et al., 2016)	CVaR	-	-
Present study	Mean-CVar	RNN, LSTM	FAHP,CoCoSo

Methodology

The present study is quantitative research. Quantitative research is a systematic and scientific research method based on a collection of information and data from the studied phenomena. In this research method, after classifying and preparing information for processing, statistical and mathematical computational techniques are used to model the behavior of phenomena. In terms of purpose, this research is considered applied research. The population of interest used in the present study is all companies that have been listed on the Tehran Stock Exchange before May 2016. Therefore, the close stock price of the mentioned companies during the period from the beginning of May 2016 to the beginning of May 2021 and also the fundamental and technical data used in the research were extracted from other information sources, such as Financial Information Processing of IRAN (FIPIRAN), Tehran Securities Exchange Technology Management Co (TSETMC), and Comprehensive Database Of All Listed Companies (CoDAL), etc. The approach used to approximate the CVaR methodology is parametric by the Variance-Covariance method. Parametric approaches involve parameterizing price behavior. This study extends the CVaR minimization approach

(Uryasev, 2000) to other classes of problems with CVaR functions. The problem of stock portfolio optimization. This study consists of four sections which are described below.

- Section 1: Selecting the top ten industries of all active industries in the Tehran Stock Exchange during the period from the beginning of May 2016 to the beginning of May 2021 using AHP.

The Steps of Chang's Fuzzy Analytic Hierarchy Process (Fuzzy AHP) Method are as follows:

Step 1: Development of the problem hierarchy. The first step in all Multi-Criteria Decision Making (MCDM) methods is to draw a decision tree. Step2: Defining fuzzy numbers. Step3: obtaining the fuzzy comparison matrices using fuzzy numbers. The fuzzy comparison matrix is as follows:

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \dots & X_{2n} \\ \dots & \dots & \dots & \dots \\ X_{n1} & \tilde{a}_{n2} & \dots & 1 \end{bmatrix}$$

Step 4: Calculate S_k for each row of the comparison matrices. S_k is calculated from the following formula:

$$S_k = \sum_{j=1}^{n} M_{kI} \times \left[\sum_{K=1}^{m}\sum_{j=1}^{n} M_{ij}\right]^{-1}$$

Where k represents the row number and j denotes the column number. M represents the triangular fuzzy numbers inside the Comparison matrix

Step 5: Calculate the magnitude of S_k with respect to each other. Consider two triangular numbers M_1 and M_2, which are drawn in Figure 1. In general, if $M_1=(I_1,m_1,u_1)$ and $M_2=(I_2,m_2,u_2)$ are two triangular fuzzy numbers,

$$M_1+M_2= (I_1+I_2,m_1+m_2,u_1+u_2)$$

$$M_1\times M_2= (I_1\times I_2,m_1\times m_2,u_1\times u_2)$$

$$M_1^{-1}=\left(\frac{1}{u_1},\frac{1}{m_1},\frac{1}{I_1}\right) \quad , M_2^{-1}=\left(\frac{1}{u_2},\frac{1}{m_2},\frac{1}{I_2}\right)$$

then, as shown in the following figure, the magnitude of M_1 with respect to M_2 can be defined as follows:

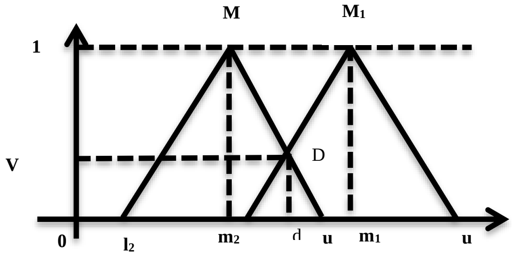


Figure 1(Chang, 1996)

$$V(M_2 \geq M_1)=hgt(M_1 \cap M_2)=\mu_{M_1}(d)=\begin{cases} 1 & \text{if } m_2 \geq m_1 \\ 0 & \text{if } l_1 \geq u_2 \\ \dfrac{l_1-u_2}{(m_2-u_2)-(m_1-l_1)} & \text{otherwise} \end{cases}$$

Step 6: Compute the weight of the criteria and alternatives in the comparison matrix. The following formula can be used for this purpose:

$$W'(X_i)=Min\{V(S_i \geq S_K)\}, \quad K=1, 2,\dots, n , K \neq 1$$

STEP 7: Calculate the final weight vector

To calculate the final weight vector, the calculated weight vector in the previous step should be normalized, then:

$$W'=\left[W'(C_1),W'(C_2),\dots,W'(C_n)\right]^T$$

In this section, the comment of capital market experts was collected using a questionnaire, then with MATLAB and weighting the criteria affecting the value of industries and normalization of weights, the top ten industries were selected from all active industries on Tehran Stock Exchange.

Normalization

After extracting stock data of top industries, because a neural network cannot receive large, heterogeneous data, we use the Min-Max normalization method to transfer all features to the range of zero to one.

$$x=\frac{x-\min(x)}{\max(x)-\min(x)}$$

- Section 2: Using LSTM to stock price forecasting.

Long Short-Term Memory (LSTM) is a type of Recurrent Neural Network (RNN). In general, an artificial neural network consists of three layers: Input layer, Hidden layer, Output layer. In LSTM, input layers are replaced by LSTM cells. An LSTM cell includes an input gate, cell state, forget gate, and output gate that can control the input flow of information. Gates also specifies which data in the sequence are important and need to be maintained, and which data do not need to be maintained and should be deleted. With this function, the network passes important information along the sequence chain to obtain the desired output. This cell also contains the sigmoid layer, the hyperbolic tangent layer (tanh). The gates and their function are as follows:

Input gate: The input gate consists of input data. Using this gate can get which amount of input should be used to change the memory. The sigmoid layer or layer of forget gate: Through this gate Can be identified which information should be removed from the cell. This is done using the sigmoid layer. This function gives each of the numbers in the cell state the number zero or one. 0 means delete and 1 means hold. Due to the tanh function, the value of new information will be between -1 and 1. This function makes decisions about the importance of information. Cell state: It runs throughout the network and is able to add or remove information with the help of gates. This gate determines the value of the next hidden state and contains information on previous inputs. In this gate, the sigmoid function decides that the values between zero and one are allowed to enter the cell. Figure 2 is an example of LSTM.

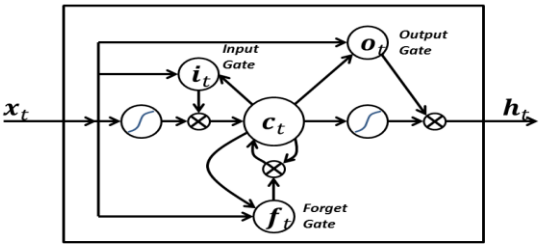


Figure 2 LSTM Recurent Neural Network (Chen et al., 2015)

In this section, we first normalized the data affecting the stock prices of companies active in selected industries from the first section, using the Min-Max formula in the range of 0 to 1, then the stock price is forecasted in the specified time periods, using Python and LSTM.

- Section 3: Stock Selection Using the CoCoSo

The steps of the Combined Compromise Solution are as follows (Yazdani et al., 2019)

(1) The initial decision-making matrix is determined as shown below:

$$\begin{bmatrix} X_{11} & X_{12} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & X_{2n} \\ \dots & \dots & \dots & \dots \\ X_{m1} & X_{m2} & \dots & X_{mn} \end{bmatrix}; \qquad i=1,2\dots,m; \quad j=1,2,\dots,n$$

X_{mn} is the evaluation of them based on the n, which can be based on both verbal expressions and real data. (Quantitative). Verbal expressions can be based on a spectrum of 5 or 9. Using the opinion of experts, the weight of the criteria was determined using Shannon's entropy method. The effective criteria for stock selection are shown in Figure 3.

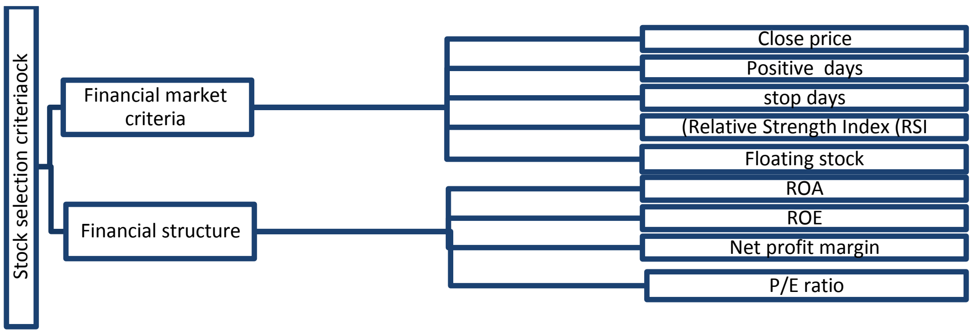


figure 3 Hierarchical structure of stock valuation and selection

(2) The normalization of criteria values is accomplished based on compromise normalization equation (Zeleny, 1973):

$$r_{ij}=\frac{X_{ij}-\underset{i}{\min}X_{ij}}{\underset{i}{\max}X_{ij}-\underset{i}{\min}X_{ij}}$$

$$r_{ij}=\frac{\underset{i}{\min}X_{ij}-X_{ij}}{\underset{i}{\max}X_{ij}-\underset{i}{\min}X_{ij}}$$

(3) Computed value of Weighted Sum Method (WSM) and Weighted Product Method (WPM): The total of the weighted comparability sequence and the whole of the power weight of comparability sequences for each alternative sum of the weighted comparability Sequence and also an amount of the power weight of comparability sequences for each alternative as S_i and P_i, respectively (Yazdani et al., 2019). W_i is the weight of the criteria. Si is derived from the SAW method and Pi is derived from the WASPAS method.

$$S_i=\sum_{j=1}^{n}(w_j\, r_{ij})$$

$$P_i=\sum_{j=1}^{n}(r_{ij})^{w_j}$$

(4) Determining the weight of each alternative: In this step, the relative weights of the alternative are computed based on 3 strategies. Three appraisal score strategies are used to generate relative weights of other options, which are derived using Formulas (15)–(17):

$$k_{ia}=\frac{P_i+S_i}{\sum_{i=1}^{m}(P_i+S_i)}$$

$$k_{ib}=\frac{S_i}{\min S_i}+\frac{P_i}{\min P_i}$$

$$k_{ic}=\frac{\lambda(S_i)+(1-\lambda)(P_i)}{(\lambda\max S_i+(1+\lambda)\max P_i)},\quad 0\leq\lambda\leq1$$

In formula 13 λ is determined by the decision-maker. Its value is usually 0.5. However, the flexibility and stability of the CoCoSo method can depend on other factors.

(5) Determine the final ranking of the alternative :In this step, the final rank of alternative is calculated using Formula 14. In fact, Formula 14 represents the geometric mean and arithmetic mean of the three strategies of the fourth step. The rank (K) for any larger alternative indicates the superiority of that alternative.

$$k_i=(k_{ia}k_{ib}k_{ic})^{\frac{1}{3}}+(k_{ia}+k_{ib}+k_{ic})$$

In this section, the top ten stocks were selected for each of the Short-term, Mid-month and Long-term portfolios, Provided that there is one stock in the portfolio from each selected industry.

- Section 4: Drawing an efficient frontier based on the Mean-CVaR model.

VaR and CVaR

VaR is one of the well-known downside risk measures, which measures the worst expected loss with a specified probability (e.g. 95%) over a given period of time. In other words, with a confidence level of α-1, the VaR is the smallest number γ, so that the probability of loss L does not exceed γ, the maximum is equal to α-1 (Rahnama, 2016).

$$(X)=\{Y\in\mathbb{R}:P(X\geq Y)\leq 1-\alpha\}=inf\{Y\in\mathbb{R}:\Psi(x,y)>\alpha\}$$

In Formula 19, Ψ (x, γ) represents the cumulative distribution function. One of the limitations of VaR is not considering the property of diversification. This means that VaR of the portfolio is greater than the total VaR of the assets that make up the portfolio. This concept is shown in Formula 20.

VaR□X+Y□□□□VaR□X□+VaR□Y□

Also, VaR is non-convex and non-smooth and has multiple local minimums, which makes the problem difficult to seek the global minimum. Another criticism of VaR is that it pays no attention to the magnitude of losses beyond the VaR value. Due to the above-mentioned shortfalls of VaR a modified alternative named CVaR has been introduced (Rahnama, 2016)

CVaR, also known as the expected shortfall, is a risk assessment measure that quantifies the amount of tail risk an investment portfolio has. CVaR is derived by taking a weighted average of the "extreme" losses in the tail of the distribution of possible returns, beyond VaR cutoff point. CVaR is used in portfolio optimization for effective risk management.

For the discrete probability distribution (where v_j occurs with the probability p_j for j=1,2,…n), CVaR is formulated as Formula 17. The decision variable is denoted by x and the random events are denoted by the vector v. p (v) is a function of probability density. F (x, v) represents the loss function proportional to the decision vector x. The loss function is a random variable and the property of a set of real numbers (Cornuejols & Tütüncü, 2006):

$$CVaR_{\propto}(X)=\frac{1}{1-\propto}\int_{j:f(x,v)\geq VaR_{\propto}(x)} p_j f(x,v_j)$$

In order to optimize the Stock portfolio with CVaR Formula 21 was changed fo Formula 22.

$$F_{\alpha}(x,\gamma):=\gamma+\frac{1}{1-\propto}\int_{f(x,v)\geq\gamma}(f(x,v)-\gamma)p(v)d(v)$$

$$\text{or}\quad F_{\alpha}(x,\gamma):=\gamma+\frac{1}{1-\propto}\int (f(x,v)-\gamma)^{+}\,p(v)d(v)$$

According to Formula 23, in order to minimize the CVaR, it is necessary to minimize the function F_α (x, γ).

$$\text{Min}\ (x) = \min(x,\gamma)$$

Therefore, to solve the stock portfolio optimization problem using the Mean-CVaR method, Formula 24 is used with the introduction of artificial variables z_i (Cornuejols & Tütüncü, 2006)

$$\text{Min}\ \ y+\frac{1}{(1-\propto)S}\sum_{i=1}^{s} z_i$$

S.t.

$$z_i\geq(f(x,v_i)-y) \qquad \forall\ i=1,\cdots,S$$

$$z_i\geq 0 \qquad \forall\ i=1,\cdots,S$$

$$x\in X$$

So, in this section, using MATLAB, the optimal weights of stock portfolios, using the Mean-CVaR was obtained and the efficient frontier was drawn according to the optimal weights.

In Figure 4, the research methodology is summarized.

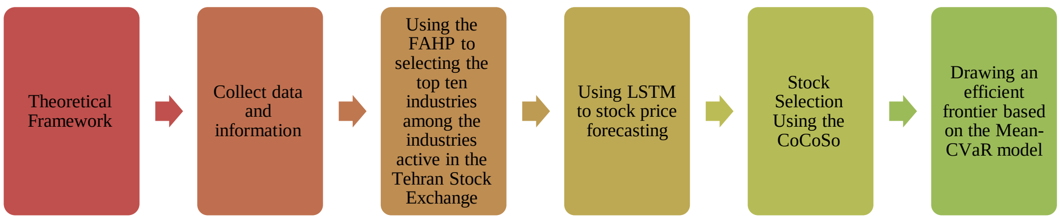


figure 4 research methodology

Data analysis and findings

The results of this study, in relation to providing a Hybrid Model for Portfolio Optimization Based on Stock Price Forecasting with LSTM Recurrent Neural Network using Multi-Criteria Decision Making Techniques and Mean-CVaR during the period from the beginning of May 2016 to the beginning of May 2021, is as follows:

- Selecting industries using FAHP

After collecting the data from the industry scoring questionnaire, the data were normalized. Based on the weights assigned to the criteria, the top ten industries are selected using FAHP. The results are given in Table 2.

Table 2Selected industries using FAHP

Top 10 Industries	
Manufacture of Motor Vehicles(MM)	Investments (I)
Food & Beverage Products (F&B)	Oil Products (OP)
Manufacture of Pharmaceuticals (MP)	Cement, Lime & Plaster (C,L&P)
Base metals (BM)	chemical products (Cp)
Insurance (In)	Banks and credit institutions (B&C)

- stock Price forecasting with LSTM

In the next section, the stock prices of companies active in top ten industries from the first section are forecasted in the specified time periods, using Python and LSTM. Based on the forecasted price, stock returns are calculated in Short-term, Mid-term, and Long-term time horizons. The results are shown comparatively in Figures 6 and 7.

- Stock selection with CoCoSo

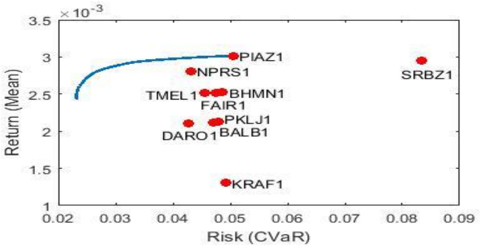
In this study, investors' preferences are considered based on the investment time horizon, so by collecting questionnaire information received from experts and then the normalization the weight of the criteria, 10 stocks in each of the short-term (one-month), mid-term (six-month), and long-term (one-year) time horizons are selected using Shannon entropy. The stokes selected using CoCoSo for 3 stock portfolios are as shown in Table 2.

Table 3 Selected stocks using CoCoSo

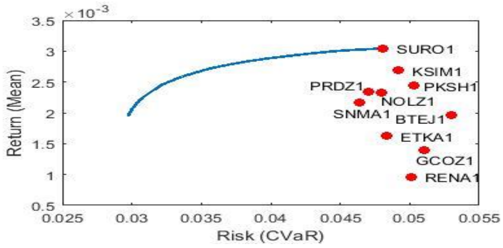
Company Symbols			Industry's Group
Long - term	Mid-term	Short-term	
GOST1	RENA1	BHMN1	MM
SKBV1	SNMA1	TMEL1	I
FOLD1	KSIM1	FAIR1	BM
GCOZ1	GCOZ1	PIAZ1	F&B
PNBA1	NOLZ1	NPRS1	OP
BSRZ1	PRDZ1	PKLJ1	Cp
EXIR1	PKSH1	DARO1	MP
KHOC1	SURO1	SRBZ1	CL&p
BANK1	BTEJ1	KRAF1	B&C
ETKA1	ETKA1	BALB1	In

Determining selected stock weights based on Mean-CVaR model to create an optimal stock portfolio

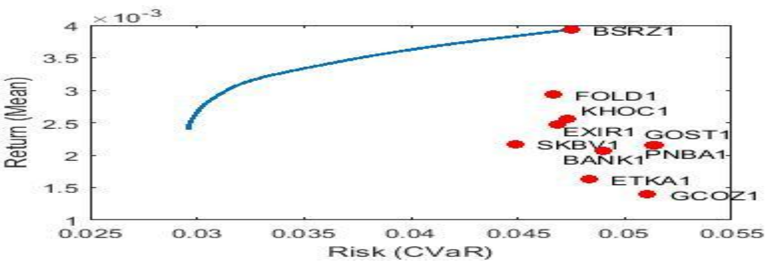
Using the Last price of selected stocks in the study period and using MATLAB, the returns and risk of stock portfolios in 100 strategies were determined and the optimal weights for short-term, mid-term and long-term stock portfolios based on Mean-CVaR model at 95% confidence level. The efficient frontier is drawn based on one hundred strategies, as shown in Figures 5 for the three portfolios. Figures 5 show that, given the efficiency frontier created for the three portfolios, Investors can buy selected stocks based on the degree of risk tolerance and expectation of return. According to the 100th strategy, at the 95% confidence level, risk-averse investors can, in order to receive the highest rate of return against the highest level of risk tolerance, in the short-term PIAZ1's stock, in the mid-term SURO1's stock, and in the long-term Buy BSRZ1's stock.



Short-term efficient frontier



Mid-term efficient frontier



Long- term efficient frontier

Figure 5 Efficient FRONTIERS based on Mean-CVaR

In Figure 6, all possible strategies for the three stock portfolios, short-term, mid-term, and long-term are shown with blue dots. Points that are close to the efficient frontier are more optimal

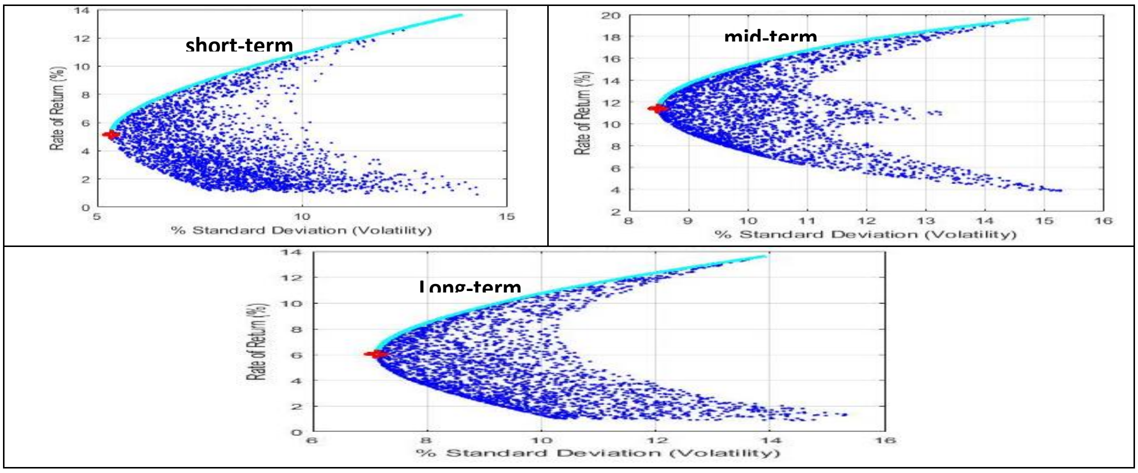


Figure 6 All possible strategies for the three stock portfolios

Comparing Returns

The real and forecasted (expected) returns of selected stocks against their industry returns are plotted in Figure 7. The results show the efficiency of the model presented in this research.

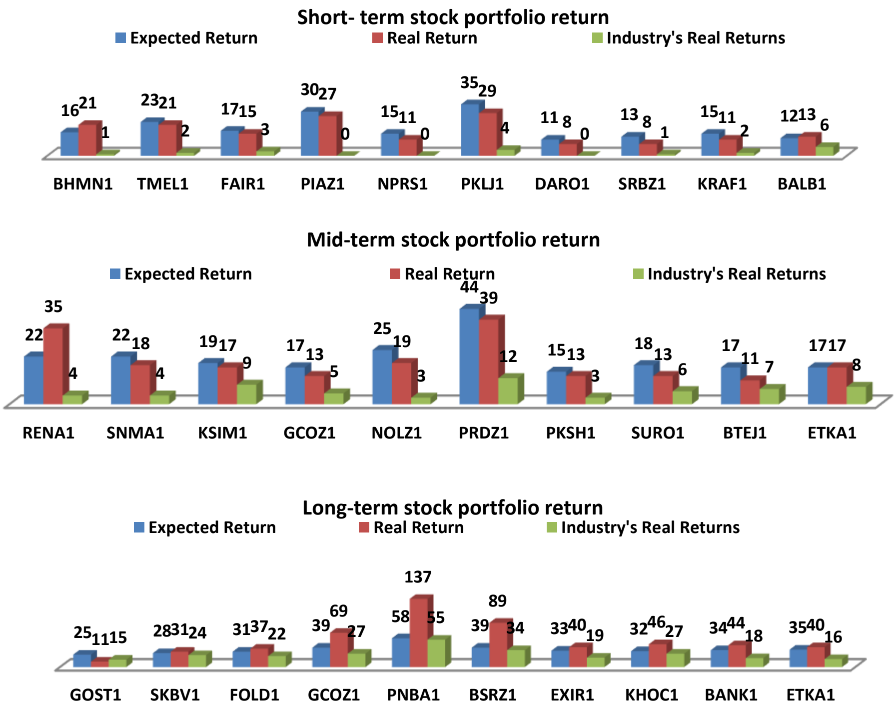


Figure 7 Comparing returns

Discussion and conclusion

Investors in financial markets are looking for a portfolio that is most efficient in terms of risk and return and has the highest return with a certain level of risk. Therefore, to achieve this goal, analysts use historical data on financial asset prices and information in the financial statements of companies to analyze and select the best portfolio. However, the lack of a systematic method for evaluating the basic and technical criteria and the effect of market psychological emotions on price is quite evident in traditional methods And disrupts the selection of the portfolio. To solve this problem, in the present study, by combining decision-making technique of FAHP, CoCoSo, and price forecasting with LSTM and finally creating optimal portfolios in three short-term, mid-term and long-term time horizons using Mean-CVaR model was created to help investors choose the optimal investment portfolio according to their preferences. The fundamental and technical data used in the research were extracted from other information sources, such as Financial Information Processing of IRAN (FIPIRAN), Tehran Securities Exchange Technology Management Co (TSETMC), and Comprehensive Database Of All Listed Companies (CoDAL), etc. Comparison of real data with the results of this study (Figure5) shows that the proposed model has acceptable efficiency and performance in creating an optimal stock portfolio. As can be seen, the majority of stocks return higher than the relevant selected industries. The difference between the real returns and the findings of the present study is mainly due to the decline in the Stock Exchange index from 2020 to 2021, factors affecting prices including investor preferences, political and economic issues governing current market conditions (coronavirus pandemic, presidential election and etc.). The results showed that combining decision-making methods based on fundamental and technical criteria with forecasting and then stock portfolio optimization has optimal results than traditional methods (methods that have used each of the techniques independently). Also, due to the use of objective and tangible criteria in addition to stock prices, investors will have a better understanding of the proposed method, so the proposed model is more acceptable among stock market participants.

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