Fixed cost allocation in bank branches: A network DEA approach

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ABSTRACT
Data Envelopment Analysis (DEA) approach is a mathematical method of exploring homogeneous decision-making units (DMUs). The structure of DMUs may have multiple steps, where one of the steps employs the output of another as its input. Manufacturing operations and industrial systems, in particular, use multi-stage structures. This study addresses the topic of constant cost distribution in a specific type of two-stage system. First, a set of common weights and sized-based allocations are performed. Eventually, a min-max approach is established to decrease the difference between efficient and size-based allocations. The proposed method for determining fixed cost allocation in bank branches has been performed on 37 branches of Iranian banks.

Keywords:
Data envelopment analysis (DEA); fixed-cost allocation; two-stage process; operation size; min-max model

1. Introduction

Data envelopment analysis (DEA) (Charnes et al. 1978) has been the subject of extensive research and empirical applications (Emrouznejad et al., 2008; Emrouznejad and Yang, 2018) and has been shown to successfully efficiency of decision-making units (DMUs), among other applications. DEA can be particularly useful in evenly distributing the fixed costs across DMUs, a process known as fixed-cost allocation. The money charged to establish sharable facilities for an organization's divisions is regarded as fixed-cost. Cook and Kress (1999) were the pioneers to implement fixed cost allocation, designing the allocation strategy using the core concepts of performance invariance and input Pareto-minimality. They realized that the assigned expenses corresponded to the current resource utilization. Cook and Zhu (2005) added an output orientation to the model and challenged the research by Cook and Kress in not meeting the efficiency invariance. They then devised an adjustment required to fulfill the performance invariance, as well as described a novel, more general allocation approach. Jahanshahloo et al. (2004) formulated a series of inequalities requirements for performance assessment of DEA-based simulations that can achieve performance invariance. They also assessed the assigned expenses associated with the proportion of a DMU's input variables to its inputs. Lin (2011a) declared the approach stated by Cook and Zhu cannot function with constraints. Lin (2011b) performed a fixed cost allocation based on the assumption that the allocation technique is representative of each DMU's proportional effectiveness and input-output ratios, with no changes in effectiveness.

Jahanshahloo et al. (2017) offered instances to show that Amirteymouri and Kordrostami's (2005) usage of shared weights to maintain efficiency invariance does not apply to the notion of efficiency invariance. They devised a fixed-cost allocation methodology based on both the concept of common weights and the theory of efficiency invariance. Furthermore, Li et al. (2017) developed an allocation of resources technique as well as a target-setting strategy based on common weights and efficiency invariance.

In the mentioned methods, it is unequivocal that no change is permitted before and after the allocation of fixed costs in the efficiency. Beasley (2003) proposed a new non-linear strategy for allocating fixed expenses that uses a number of shared weights to achieve the highest average efficiency for all DMUs. This strategy, according to Beasley (2003), may produce a fair and balanced fixed-cost distribution to DMUs, resulting in maximum efficiency. The simulations established based on this method are all nonlinear to allow for a fixed-cost allocation.

Si et al. (2013) and Li et al. (2013) suggested that using similar weights for fixed cost allocation promotes efficiency for all DMUs. Hosseinzadeh Lotfi et al. (2013) have established a number of common weights and an optimal planning technique for fixed-cost allocation that assures the efficiency of all DMUs.

A fixed-cost allocation approach can be used with other DEA ideas. Du et al. (2014) presented an iterative approach based on cross-efficiency for usage in fixed-cost allocation as well as input resource allocation. Another method of fixed-cost allocation suggested by Li et al. (2018) was DEA-game cross-efficiency. This approach uses common weights to determine the specific allocation. Hosseinzadeh Lotfi et al. (2007) investigated a fixed-cost allocation in a fuzzy environment with total shared expenses, fuzzy inputs, and outputs. Pendharkar proposed a composite genetic algorithm based on DEA (2017).

In traditional DEA simulations, the interior structure of units is not addressed, leading them to have a “black box” approach. This led to the advancement of NDEA (analysis of the network data envelopment) models that are applicable to evaluate multi-stage networks. All of the above-mentioned studies regard every DMUs as a black box. There have been attempts at fixed-cost allocation in the context of a system. Bi et al. (2011) introduced a two-stage, parallel-structure model with the three criteria of optimizing the performance, improving the performance of poorest-performing DMUs, and common weights. Another network-based fixed-cost allocation simulation having a parallel structure was established by Hadi Vencheh et al. (2014). Xiong et al. (2017) allocated resources for a multi-directional parallel system. Yu and Chen (2016) investigated centrally controlled resource allocation and dissipation tolerance in a manufacturing system with two steps. The integration of the reduction of unwanted outputs and the reduction of energy inputs was performed in the system, and the adaptation of min-max was designed for the ultimate allocation and minimization strategy. Yu and others (2016) proposed a DEA two-stage fixed-cost problem with regards to the Du et al.’s (2014) cross-efficiency repeating methodology. The use of this strategy demonstrates the varying degrees of performance among DMUs.

A number of DEA simulations and secondary purposes were proposed by Zhu et al. (2017) to allocate fixed costs and common resources in two-stage networks. However, these models failed to determine above-zero expenses for subphases and, in addition, were not able to make most of the DMUs efficient. The issues of resource allocation and centralized fixed-cost allocation were studied by Ding et al. (2018) using heterogeneity technology. Ding et al. (2019) suggested extending it to general networks with two levels, while Ding et al. (2020) implemented heterogeneity technology to the general two-level network. This problem was further explored by Zhu et al. (2019). An et al. (2020) used the efficiency invariance concept to handle fixed-cost allocation in basic two-stage networks before expanding its applicability to generalized two-stage networks. Lin and Chen (2020) focused on the fixed-cost division issue as it related to the initial intake among decision-making divisions. Chu and Jiang (2019) investigated the fixed-cost allocation according to their competence. Chu et al. (2020) employed a new method for fixed-cost allocation within DMUs in a two-stage network. Their strategy included the leader-follower paradigm as well as the amount of satisfaction obtained through the bargaining game framework. Ebdali and Fallah Nejad (2020) suggested a bargaining framework with non - discretionary inputs to assess the effectiveness of DMUs in a network with two levels. Li et al. (2019) utilized DEA for fixed cost-allocation in a decentralized context. Recently, Li et al. (2021) suggested considering the performance of DMUs in fixed-cost allocation.

Researching banking issues is very important because banks can highly affect the economic growth of countries. There are now many studies that have been conducted on the banking industry using the data envelopment analysis technique. Studies in this field are mainly classified into two groups; studies on the branches of banks of one country and evaluation and comparison of banks of different countries. Kaffash and Marra (2017) examined the DEA method and its applications in financial services. They reviewed 620 articles published in journals on the Science Database website and they also analyzed DEA in the banking industry, money market funds, and the insurance industry. An et al. (2020) investigated the issue of fixed cost allocation for two-stage networks using participatory relationships and data envelopment analysis, and they used the analysis results for Chinese commercial banks. In another article, An et al. (2021) investigated the issue of fixed cost allocation for a two-stage network using non-participatory relationships and examined the results for bank data. Li et al. (2021) also investigated fixed cost allocation in data envelopment analysis based on performance ranking and implemented the proposed approach using bank data. Other studies that have been conducted in the field of DEA, social security, pensions, and stock exchanges are as follows: Ravanshad et al.(2019), Ashrafi et al.(2018), Alinezhad (2018) and Pargar et al.(2022)

This study employed the method introduced by Li et al. (2019) to develop a model involving the idea of size-based fixed-cost allocation. An optimal planning model is used in this case to solve the issue of lack of consistency of allocation using this technique with the efficiency of allocation for the intended network. This research also includes a technology that is based on the concept of continuous returns to scale (CRS).

This article's novel contributions are as follows:

1) A novel fixed-cost allocation issue for a two-stage network.
2) The efficiency attained in the final allocation outcomes for all DMUs and sub-phases relying on common weights.
3) Inclusion of DMU operation size in allocation, with the consequent uniformity of the allotted cost with the usage of input and output production in terms of size.
4) Suggesting a min-max model ensure a minimum deviation for all units to achieve the fixed cost allocation.

The rest of the paper consists of the following sections. Section two presents the basic and elementary concepts of DEA. Section three discusses the application of the fixed cost allocation model on a specific network process, achieving an efficiency of one for the entire network and stages. Also, an optimal planning model is used to minimize the deviation to achieve a fixed cost allocation. The fourth section examines the practical example of allocating fixed cost on of Iranian Bank . Finally, in Section 5, conclusions and recommendations are presented.

2. Basic concepts

If n DMUs are generated for any DMU_j (j=1,..., n) by input $X_j=(x_{1j},..., x_{mj})$ and output $Y_j=(y_{1j},..., y_{sj})$ vectors, the performance of DMU_d (d=1,..., n) may be assessed using the classic approach CCR (1978):

$$E_d^* = Max \frac{\sum_{r=1}^{s} \mu_r y_{rd}}{\sum_{i=1}^{m} v_i x_{id}}$$

$$s.t \quad \frac{\sum_{r=1}^{s} \mu_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad j = 1, \ldots, n$$

$$\mu_r, v_i \ge 0 \quad , \quad r = 1, \ldots, s \quad , \quad i = 1, \ldots, m. \ (1)$$

This model has a fractional non-linear structure linearized through Charles-cooper transformation.

$$E_d^* = Max \sum_{r=1}^{s} u_r y_{rd}$$

$$s.t \quad \sum_{i=1}^{m} v_i x_{id} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 0, \ldots, n$$

$u_r, v_i \geq 0 \quad , \quad r = 1, \ldots, s \quad , \quad i = 1, \ldots, m. \ (2)$

An optimal weight (u_r^{d*}, v_i^{d*}) can be derived from this model. Here, the formula $E_d^* = \sum_{r=1}^{s} u_r^{d^*} y_{rd}$ can determine the optimal efficiency for each DMU_d (d=1, …, n). The value of E_d^* varies between zero and one. DMU_d is DEA-efficient if its highest performance score is equal to one. DMU_d , on the other hand, is regarded DEA inefficient when the performance level is below one.

3. Evaluation of performance via fixed-cost allocation as well as network structure

The system sources access the first sub-phase, and the result of such a subphase accesses the two components as inputs in the subsequent subphase if the DMUs in Figure 1 exist in a two-phase network.

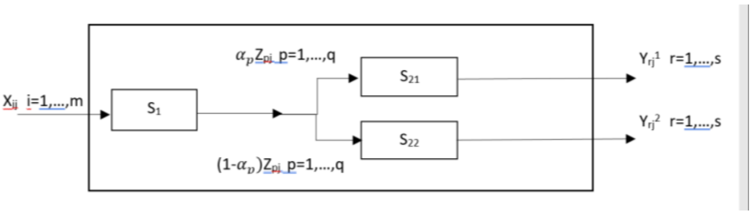

Figure 1- DMU_j two-stage network

A fixed-cost R can also be assumed for DMUs that all of them should cover. As DMUj (j=1,..., n) receives the non-negative cost of Rj, the following is achieved:

$$\sum_{j=1}^{n} R_j = R \qquad R_j \geq 0 \quad j = 1, \ldots, n \ (3)$$

R_{1j} and R_{2j} in the preceding equation provide the fixed-cost allocation expense of the original and subsequent phases, resulting in:

$$R_j = R_{1j} + R_{2j} \qquad R_{1j}\ ,\ R_{2j} \geq 0 \quad j = 1, \ldots, n \ (4)$$

The fixed-cost allocation of the second stage of the network can also be written as:

$$R_{2j} = R_{21j} + R_{22j} \qquad R_{21j}\ ,\ R_{22j} \geq 0 \quad j = 1, \ldots, n \ (5)$$

According to Chen et al. [11] [13], the network's overall performance incorporating fixed-cost allocation for DMUs constructed as illustrated in figure (1) may be expressed as follows:

$$e_d^* = \max(w_1 e_{1d} + w_2^1 e_{2d}^1 + w_2^2 e_{2d}^2)$$

$$s.t \quad e_{1j} = \frac{\sum_{p=1}^{q} \varphi_p z_{pj}}{\sum_{i=1}^{m} v_i x_{ij} + v_{m+1} R_{1j}} \leq 1 \quad j = 1, \ldots, n$$

$$e_{2j}^1 = \frac{\sum_{r=1}^{s} \mu_r^1 y_{rj}^1}{\sum_{p=1}^{q} \varphi_p \alpha_p z_{pj} + v_{m+1} R_{21j}} \leq 1 \quad j = 1, \ldots, n$$

$$e_{2j}^2 = \frac{\sum_{r=1}^{s'} \mu_r^2 y_{rj}^2}{\sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pj} + v_{m+1} R_{22j}} \leq 1 \quad j = 1, \ldots, n$$

$$\sum_{j=1}^{n} \left(R_{1j} + R_{21j} + R_{22j}\right) = R \qquad R_{1j}, R_{21j}, R_{22j} \geq 0 \quad j = 1, \ldots, n$$

$$\mu_r, \varphi_p, v_i \geq 0 \quad , \quad v_{m+1} > 0 \quad , \quad r = 1, \ldots, s \quad , \quad p = 1, \ldots, q \quad , \quad i = 1, \ldots, m \ (6)$$

Where W_1 denotes the weight of the first stage, W_2^1 of the initial subphase, and W_2^2 the weight of the subsequent subphase of the process illustrated in figure (1), where $W_1 + W_2^1 + W_2^2 = 1$. It can be written as:

$$W_1 = \frac{\sum_{i=1}^{m} v_i x_{id} + v_{m+1} R_{1d}}{\sum_{i=1}^{m} v_i x_{id} + v_{m+1} R_{1d} + \sum_{p=1}^{q} \varphi_p \alpha_p z_{pd} + v_{m+1} R_{21d} + \sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pd} + v_{m+1} R_{22d}}$$

$$W_2^1 = \frac{\sum_{p=1}^{q} \varphi_p \alpha_p z_{pd} + v_{m+1} R_{21d}}{\sum_{i=1}^{m} v_i x_{id} + v_{m+1} R_{1d} + \sum_{p=1}^{q} \varphi_p \alpha_p z_{pd} + v_{m+1} R_{21d} + \sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pd} + v_{m+1} R_{22d}} \ (7)$$

$$W_2^2 = \frac{\sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pd} + v_{m+1} R_{22d}}{\sum_{i=1}^{m} v_i x_{id} + v_{m+1} R_{1d} + \sum_{p=1}^{q} \varphi_p \alpha_p z_{pd} + v_{m+1} R_{21d} + \sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pd} + v_{m+1} R_{22d}}$$

When these formulas are entered into target function (6), the result is as follows:

$$e_d = \frac{\sum_{p=1}^{q} \varphi_p z_{pd} + \sum_{r=1}^{s} \mu_r^1 y_{rd}^1 + \sum_{r=1}^{s'} \mu_r^2 y_{rd}^2}{\sum_{i=1}^{m} v_i x_{id} + v_{m+1} R_{1d} + \sum_{p=1}^{q} \varphi_p \alpha_p z_{pd} + v_{m+1} R_{21d} + \sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pd} + v_{m+1} R_{22d}} \quad (8)$$

Therefore, model (6) can be re-expressed as:

$$e_d^* = Max \frac{\sum_{p=1}^{q} \varphi_p z_{pd} + \sum_{r=1}^{s} \mu_r^1 y_{rd}^1 + \sum_{r=1}^{s'} \mu_r^2 y_{rd}^2}{\sum_{i=1}^{m} v_i x_{id} + v_{m+1} R_{1d} + \sum_{p=1}^{q} \varphi_p \alpha_p z_{pd} + v_{m+1} R_{21d} + \sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pd} + v_{m+1} R_{22d}}$$

$$s.t \quad e_{1j} = \frac{\sum_{p=1}^{q} \varphi_p z_{pj}}{\sum_{i=1}^{m} v_i x_{ij} + v_{m+1} R_{1j}} \le 1 \qquad j = 1, \dots, n$$

$$e_{2j}^1 = \frac{\sum_{r=1}^{s} \mu_r^1 y_{rj}^1}{\sum_{p=1}^{q} \varphi_p \alpha_p z_{pj} + v_{m+1} R_{21j}} \le 1 \qquad j = 1, \dots, n$$

$$e_{2j}^2 = \frac{\sum_{r=1}^{s'} \mu_r^2 y_{rj}^2}{\sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pj} + v_{m+1} R_{22j}} \le 1 \qquad j = 1, \dots, n$$

$$\sum_{j=1}^{n} \left(R_{1j} + R_{21j} + R_{22j}\right) = R \qquad R_{1j}, R_{21j}, R_{22j} \ge 0 \quad j = 1, \dots, n$$

$$\mu_r, \varphi_p, v_i \ge 0 \quad , \quad v_{m+1} > 0 \quad , \quad r = 1, \dots, s \quad , \quad p = 1, \dots, q \quad , \quad i = 1, \dots, m (9)$$

Model (9) is a fractional programming model which is very difficult to compute. Yet it can be linearized using Charnes-Cooper transformation (1962), as below:

$$e_d^* = Max(\sum_{p=1}^{q} \phi_p z_{pd} + \sum_{r=1}^{s} u_r^1 y_{rd}^1 + \sum_{r=1}^{s'} u_r^2 y_{rd}^2)$$

$$s.t \quad \sum_{i=1}^{m} v_i x_{id} + v_{m+1} R_{1d} + \sum_{p=1}^{q} \varphi_p \alpha_p z_{pd} + v_{m+1} R_{21d} + \sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pd} + v_{m+1} R_{22d} = 1$$

$$\sum_{p=1}^{q} \phi_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} - v_{m+1} R_{1j} \le 0 \qquad j = 1, \dots, n$$

$$\sum_{r=1}^{s} \mu_r^1 y_{rj}^1 - \sum_{p=1}^{q} \phi_p \alpha_p z_{pj} - v_{m+1} R_{21j} \le 0 \quad j = 1, \dots, n$$

$$\sum_{r=1}^{s'} u_r^2 y_{rj}^2 - \sum_{p=1}^{q} \phi_p (1-\alpha_p) z_{pj} - v_{m+1} R_{22j} \le 0 \qquad j = 1, \dots, n$$

$$\sum_{j=1}^{n} \left(R_{1j} + R_{21j} + R_{22j}\right) = R \qquad R_{1j}, R_{21j}, R_{22j} \ge 0 \quad j = 1, \dots, n$$

$$u_r, \phi_p, v_i \ge 0 \quad , \quad v_{m+1} > 0 \quad , \quad r = 1, \dots, s \quad , \quad p = 1, \dots, q \quad , \quad i = 1, \dots, m (10)$$

It should be noted that model (10) is a non-linear programming model if the multiplication of $\phi_p \alpha_p z_{pj}$, $v_{m+1} R_{1j}, v_{m+1} R_{21j}$ and $v_{m+1} R_{22j}$ exists in the restrictions. By transforming the variables as $v_{m+1} R_{22j} = r_{22j}$, $v_{m+1} R_{21j} = r_{21j}$, $v_{m+1} R_{1j} = r_{1j}$ (for j=1, …, n) and $\phi_p \alpha_p z_{pj}$, the model can be solved this way:

$$e_d^* = \max(\sum_{p=1}^{q} \phi_p z_{pd} + \sum_{r=1}^{s} u_r^1 y_{rd}^1 + \sum_{r=1}^{s'} u_r^2 y_{rd}^2)$$

$$s.t \quad \sum_{i=1}^{m} v_i x_{id} + r_{1d} + \sum_{p=1}^{q} \omega_p z_{pd} + r_{21d} - \sum_{p=1}^{q} \omega_p z_{pd} + \sum_{p=1}^{q} \phi_p z_{pd} + r_{22d} = 1$$
$$\sum_{p=1}^{q} \phi_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} - r_{1j} \leq 0 \qquad j = 1, \dots, n$$
$$\sum_{r=1}^{s} u_r^1 y_{rj}^1 - \sum_{p=1}^{q} \omega_p z_{pj} - r_{21j} \leq 0 \qquad j = 1, \dots, n$$
$$\sum_{r=1}^{s'} u_r^2 y_{rj}^2 + \sum_{p=1}^{q} \omega_p z_{pj} - \sum_{p=1}^{q} \phi_p z_{pj} - r_{22j} \leq 0 \qquad j = 1, \dots, n$$
$$\sum_{j=1}^{n} (r_{1j} + r_{21j} + r_{22j}) = v_{m+1} R \qquad r_{1j}, r_{21j}, r_{22j} \geq 0 \quad j = 1, \dots, n$$
$$u_r, \phi_p, v_i \geq 0 \quad , \quad v_{m+1} > 0 \quad , \quad r = 1, \dots, s \, , \quad p = 1, \dots, q \, , \quad i = 1, \dots, m (11)$$

The simplified model will be:

$$e_d^* = \max(\sum_{p=1}^{q} \phi_p z_{pd} + \sum_{r=1}^{s} u_r^1 y_{rd}^1 + \sum_{r=1}^{s'} u_r^2 y_{rd}^2)$$
$$s.t \quad \sum_{i=1}^{m} v_i x_{id} + r_{1d} + \sum_{p=1}^{q} \phi_p z_{pd} + r_{21d} + r_{22d} = 1$$
$$\sum_{p=1}^{q} \phi_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} - r_{1j} \leq 0 \qquad j = 1, \dots, n$$
$$\sum_{r=1}^{s} u_r^1 y_{rj}^1 - \sum_{p=1}^{q} \omega_p z_{pj} - r_{21j} \leq 0 \qquad j = 1, \dots, n$$
$$\sum_{r=1}^{s'} u_r^2 y_{rj}^2 + \sum_{p=1}^{q} (\omega_p - \phi_p) z_{pj} - r_{22j} \leq 0 \qquad j = 1, \dots, n$$
$$\sum_{j=1}^{n} (r_{1j} + r_{21j} + r_{22j}) = v_{m+1} R \qquad r_{1j}, r_{21j}, r_{22j} \geq 0 \quad j = 1, \dots, n$$
$$u_r, \phi_p, v_i \geq 0 \quad , \quad v_{m+1} > 0 \quad , \quad r = 1, \dots, s \, , \quad p = 1, \dots, q \, , \quad i = 1, \dots, m (12)$$

The optimum performance with fixed-cost allocation can be achieved through applying this model to individual DMUs. If for evaluating (*d*=1, …, *n*) DMU_d with the model (10), the optimal answer of $(u_r^{1d*}, u_r^{2d*}, \phi_p^{d*}, \omega_p^{d*}, v_i^{d*}, v_{m+1}^{d*}, r_{1j}^{d*}, r_{21j}^{d*}, r_{22j}^{d*})$ is

known, the proportional efficiency following expense allocation can be written as:

$$R_j^{d*} = R_{1j}^{d*} + R_{21j}^{d*} + R_{22j}^{d*} \qquad j = 1, \dots, n$$
$$R_{1j}^{d*} = \frac{r_{1j}^{d*}}{v_{m+1}^{d*}} \qquad j = 1, \dots, n$$
$$R_{21j}^{d*} = \frac{r_{21j}^{d*}}{v_{m+1}^{d*}} \qquad j = 1, \dots, n (14)$$
$$R_{22j}^{d*} = \frac{r_{22j}^{d*}}{v_{m+1}^{d*}} \qquad j = 1, \dots, n$$

$$e_d^* = \sum_{p=1}^{q} \phi_p^{d*} z_{pd} + \sum_{r=1}^{s} u_r^{1d*} y_{rd}^1 + \sum_{r=1}^{s'} u_r^{2d*} y_{rd}^2 (13)$$

Moreover, the fixed-cost allocation plan is specified as follows:

As a result, resolving the linear programming model (12) reveals the network's proportional efficiency and maximal allocation of stages (see Figure 1). The following are some of the model's characteristics:

Lemma 1: Model (12)'s optimum quantity is often either less or equal to one.

Proof: If the dth indicator is in the model's second, third, and fourth restrictions, it follows that:

$$\sum_{p=1}^{q} \phi_p z_{pd} - \sum_{i=1}^{m} v_i x_{id} - r_{1d} + \sum_{r=1}^{s} u_r^1 y_{rd}^1 - \sum_{p=1}^{q} \omega_p z_{pd} - r_{21d} + \sum_{r=1}^{s'} u_r^2 y_{rd}^2 + \sum_{p=1}^{q} (\omega_p - \phi_p)\, z_{pd} - r_{22d} \le 0$$

The following inequality may be generated by displacing certain terms on either side of the inequality:

$$\sum_{p=1}^{q} \phi_p z_{pd} + \sum_{r=1}^{s} u_r^1 y_{rd}^1 + \sum_{r=1}^{s'} u_r^2 y_{rd}^2 \le \sum_{i=1}^{m} v_i x_{id} + r_{1d} + r_{21d} + \sum_{p=1}^{q} \phi_p z_{pd} + r_{22d}$$

The right side of this inequality specifies the first constraint of the model (12) which is one. So, the inequality can be expressed as:

$$\sum_{p=1}^{q} \phi_p z_{pd} + \sum_{r=1}^{s} u_r^1 y_{rd}^1 + \sum_{r=1}^{s'} u_r^2 y_{rd}^2 \le 1$$

As a result, the objective function and the ideal value of model (12) are either equal or less than one. The lemma is accepted as correct.

Theorem 1: Model (12) has an ideal value of one at all times.

Proof: The dual linear planning model (12) is initially written as:

$$\begin{aligned}
&\min \theta \\
&s.t\,\theta x_d - \sum_j \lambda_j^1 x_j \ge 0 \\
&\theta z_d + \sum_j \lambda_j^1 z_j - \sum_j \lambda_j^3 z_j \ge z_d \\
&\theta - \lambda_d^1 + \beta \ge 0 \\
&-\lambda_j^1 + \beta \ge 0 \quad j \ne d \\
&\theta - \lambda_d^2 + \beta \ge 0 \\
&-\lambda_j^2 + \beta \ge 0 \quad j \ne d \\
&\theta - \lambda_d^3 + \beta \ge 0 \\
&-\lambda_j^3 + \beta \ge 0 \quad j \ne d \\
&\sum_j \lambda_j^2 y_j^1 \ge y_d^1 \\
&\sum_j \lambda_j^3 y_j^2 \ge y_d^2 \\
&-\beta R \ge 0 \\
&\beta, \lambda^1, \lambda^2, \lambda^3 \ge 0, \theta\, free
\end{aligned}$$

The definition $R \ge 0$ and restriction $-\beta R \ge 0$ show that $\beta \le 0$. From the fourth restriction group mentioned above ($0 \ge \beta \ge \lambda_j^1 (for\ j \ne d$), it can be concluded that $\lambda_j^1 = 0(for\ j \ne d)$. Likewise, restriction groups of sixth and eighth reveal that $\lambda_j^2 = \lambda_j^3 = 0(for\ j \ne d)$.

The second restriction indicates that $\sum_j \lambda_j^1 z_j \ge \sum_j \lambda_j^3 z_j + (1-\theta) z_d$. The tenth constraint of the dual model and Lemma 1 assert that if λ_j^3 = zero for each j, then y_d^2 becomes zero. The right side becomes greater than zero since this is inconsistent, which indicates that $\lambda^1 \ne 0$ and, therefore, $\lambda_d^1 > 0,\ \lambda_d^2 > 0,\ and\ \lambda_d^3 > 0$.

The comparable constraint on a positive factor in another issue should contain a neutral or active auxiliary factor based on the complement slackness criteria inherent in primal-dual issues. The corresponding constraints $\lambda_d^k (for\ k = 1,2,3)$ in problem (12) are, therefore, as below:

$$\sum_{p=1}^{q} \phi_p z_{pd} - \sum_{i=1}^{m} v_i x_{id} - r_{1d} = 0$$

$$\sum_{r=1}^{s} u_r^1 y_{rd}^1 - \sum_{p=1}^{q} \omega_p z_{pd} - r_{21d} = 0$$

$$\sum_{r=1}^{s'} u_r^2 y_{rd}^2 + \sum_{p=1}^{q} (\omega_p - \phi_p)\, z_{pd} - r_{22d} = 0$$

The following equation is based on the overall sum of the equations on the left side:

Moreover, the first constraint of the model (12) can be expressed as:

$$\sum_{i=1}^{m} v_i x_{id} + r_{1d} + \sum_{p=1}^{q} \phi_p z_{pd} + r_{21d} + r_{22d} = 1$$

$$\Rightarrow \sum_{p=1}^{q} \phi_p z_{pd} - 1 = -\sum_{i=1}^{m} v_i x_{id} - r_{1d} - r_{21d} - r_{22d}$$

$$\sum_{r=1}^{s} u_r^1 y_{rd}^1 + \sum_{r=1}^{s'} u_r^2 y_{rd}^2 \sum_{p=1}^{q} \phi_p z_{pd} - 1 = 0$$

$$\sum_{p=1}^{q} \phi_p z_{pd} + \sum_{r=1}^{s} u_r^1 y_{rd}^1 + \sum_{r=1}^{s'} u_r^2 y_{rd}^2 = 1$$

The equations below are obtained through the combination of the above relations:

The result is that the optimal value of model (12) is equal to one at all times. The theorem is considered true.

Definition 1: If after fixed-cost allocation, $e_d^* = 1$, then the efficiency of DMU_d (d=1, …, n) is confirmed.

Theorem 2: A unit with raw variables is efficient if it maintains its efficiency throughout the process.

Proof: The necessary requirement is apparent regarding the concept of DMU and subphase efficiency

If

$$e_{1j} = \frac{\sum_{p=1}^{q} \varphi_p z_{pj}}{\sum_{i=1}^{m} v_i x_{ij} + v_{m+1} R_{1j}} = 1 \qquad j = 1, \dots, n$$

$$e_{2j}^1 = \frac{\sum_{r=1}^{s} \mu_r^1 y_{rj}^1}{\sum_{p=1}^{q} \varphi_p \alpha_p z_{pj} + v_{m+1} R_{21j}} = 1 \qquad j = 1, \dots, n$$

$$e_{2j}^2 = \frac{\sum_{r=1}^{s'} \mu_r^2 y_{rj}^2}{\sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pj} + v_{m+1} R_{22j}} = 1 \qquad j = 1, \dots, n$$

Then:

$$e_j = w_1 e_{1j} + w_2^1 e_{2j}^1 + w_2^2 e_{2j}^2 = w_1 + w_2^1 + w_2^2 = 1$$

Thus, DMUs will be efficient.

The required condition here is the efficiency of both DMUs and the subphases following fixed-cost allocation.

Once DMUj is efficient after a fixed cost allocation, the following may be asserted:

$$e_j = w_1 e_{1j} + w_2^1 e_{2j}^1 + w_2^2 e_{2j}^2 = \frac{\sum_{p=1}^{q} \varphi_p z_{pj} + \sum_{r=1}^{s} \mu_r^1 y_{rj}^1 + \sum_{r=1}^{s'} \mu_r^2 y_{rj}^2}{\sum_{i=1}^{m} v_i x_{ij} + v_{m+1} R_{1j} + \sum_{p=1}^{q} \varphi_p \alpha_p z_{pj} + v_{m+1} R_{21j} + \sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pj} + v_{m+1} R_{22j}} = 1$$

According to this

$$\sum_{p=1}^{q} \varphi_p z_{pj} - (\sum_{i=1}^{m} v_i x_{ij} + v_{m+1} R_{1j}) = -[(\sum_{r=1}^{s} \mu_r^1 y_{rj}^1) - (\sum_{p=1}^{q} \varphi_p \alpha_p z_{pj} + v_{m+1} R_{21j})]$$

$$-[(\sum_{r=1}^{s'} \mu_r^2 y_{rj}^2) - (\sum_{p=1}^{q} \varphi_p (1-\alpha_p) z_{pj} + v_{m+1} R_{22j})] = 0$$

It should be noted that

$$0 \le w_1, w_2^1, w_2^2 \le 1, w_1 + w_2^1 + w_2^2 = 1, 0 < e_{1j}, e_{2j}^1, e_{2j}^2 \le 1$$

The modes below can thus be obtained:

1)$e_{1j} = 1. w_1 = 1,$

2)$e_{2j}^1 = 1. w_2^1 = 1,$

3)$e_{2j}^2 = 1. w_2^2 = 1,$

4)$e_{1j} = e_{2j}^1 = e_{2j}^2 = 1.$

This is the only condition where the equation $e_j = w_1 e_{1j} + w_2^1 e_{2j}^1 + w_2^2 e_{2j}^2 = 1$ is true. The reductio ad absurdum might be used to show that one of the relations is legitimate. This can also be used in other situations. If a contrary assumption is made for the first relation, i.e., $e_{1j} \neq 1$, the definition of Ws points at a contradiction.

A number of common weights is able to achieve efficiency for DMUs and subphases all at the same time. A collection of common weights that offer an efficient allocation for DMUs is shown below.

Theorem 3: At least one efficient allocation exists for the stated issue. This allocation validates the overall efficiency as well as the efficiency of the decision-making unit's components.

$$\sum_{p=1}^{q} \phi_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} - R_{1j} = 0 \qquad j = 1, \dots, n$$
$$\sum_{r=1}^{s} u_r^1 y_{rj}^1 - \sum_{p=1}^{q} \omega_p z_{pj} - R_{21j} = 0 \qquad j = 1, \dots, n$$
$$\sum_{r=1}^{s'} u_r^2 y_{rj}^2 + \sum_{p=1}^{q} (\omega_p - \phi_p)\, z_{pj} - R_{22j} = 0 \qquad j = 1, \dots, n$$
$$\sum_{j=1}^{n} \left(R_{1j} + R_{21j} + R_{22j}\right) = R \qquad R_{1j}, R_{21j}, R_{22j} \geq \epsilon \quad j = 1, \dots, n$$
$$u_r, \phi_p, v_i \geq 0 \quad , \quad r = 1, \dots, s \quad , \quad p = 1, \dots, q \quad , \quad i = 1, \dots, m (15)$$

Proof: According to theorems 1 and 2, it is clear.

The proposal below may be stated by normalizing the sizes in the approach provided by Feng et al. (2018) and relying on the research of Amirteimoori and Kordrostami (2005) and Lin (2011):

$$\hat{x}_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}} \ , \ \hat{z}_{pj} = \frac{z_{pj}}{\sum_{j=1}^{n} z_{pj}} \ , \ \hat{y}_{rj}^1 = \frac{y_{rj}^1}{\sum_{j=1}^{n} y_{rj}^1} \ , \ \hat{y}_{rj}^2 = \frac{y_{rj}^2}{\sum_{j=1}^{n} y_{rj}^2} (16)$$
$$\alpha_j = \frac{\sum_{p=1}^{q} \hat{z}_{pj} \cdot \sum_{i=1}^{m} \hat{x}_{ij}}{\sum_{i=1}^{n} \left(\sum_{p=1}^{q} \hat{z}_{pj} \cdot \sum_{i=1}^{m} \hat{x}_{ij} + \sum_{p=1}^{q} \hat{z}_{pj} \cdot \left(\sum_{r=1}^{s} \hat{y}_{rj}{}^1 + \sum_{r=1}^{s'} \hat{y}_{rj}{}^2\right)\right)} (17)$$
$$\beta_j = \frac{\sum_{r=1}^{s} \hat{y}_{rj}^1 \cdot \alpha \sum_{p=1}^{q} \hat{z}_{pj}}{\sum_{i=1}^{n} \left(\sum_{p=1}^{q} \hat{z}_{pj} \cdot \sum_{i=1}^{m} \hat{x}_{ij} + \sum_{p=1}^{q} \hat{z}_{pj} \cdot \left(\sum_{r=1}^{s} \hat{y}_{rj}{}^1 + \sum_{r=1}^{s'} \hat{y}_{rj}{}^2\right)\right)} (18)$$
$$\gamma_j = \frac{\sum_{r=1}^{s} \hat{y}_{rj}{}^2 \cdot (1-\alpha) \sum_{p=1}^{q} \hat{z}_{pj}}{\sum_{i=1}^{n} \left(\sum_{p=1}^{q} \hat{z}_{pj} \cdot \sum_{i=1}^{m} \hat{x}_{ij} + \sum_{p=1}^{q} \hat{z}_{pj} \cdot \left(\sum_{r=1}^{s} \hat{y}_{rj}{}^1 + \sum_{r=1}^{s'} \hat{y}_{rj}{}^2\right)\right)} (19)$$

Therefore, the cost share that is allocated to DMU_j (j=1, …, n) is equal to $(\alpha_j + \beta_j + \gamma_j)$.

The above ratios assign a share of the fixed cost to DMUs of the user input and the current produced output according to their size. However, this does not imply that a linear system of equations involving a set of common weights may be satisfied by such a relationship. The approach suggested by Feng et al. (2018) demonstrates that the logical solution to this problem is employing the ideal planning method using a set of deviation factors.

If C_j, D_j, and E_j are positive deviation variables, it can be stated that:

$$\left|\alpha_j R - R_{1j}\right| = C_j \quad , \quad \left|\beta_j R - R_{21j}\right| = D_j \quad , \quad \left|\gamma_j R - R_{22j}\right| = E_j (20)$$

For any DMU_j (j= 1, …, n).

The value of C_j, D_j, and E_j must be minimized to obtain the desired plan. Therefore, the maximal deviation $(E_j + D_j + C_j)$ from all DMUs is minimized. The model below can be used to address this problem.

$$Min \underset{k=1,\dots,n}{Max} (C_k + D_k + E_k)$$
$$s.t \quad \left|\alpha_j R - R_{1j}\right| = C_j \qquad j = 1, \dots, n$$
$$\left|\beta_j R - R_{21j}\right| = D_j \qquad j = 1, \dots, n$$
$$\left|\gamma_j R - R_{22j}\right| = E_j \qquad j = 1, \dots, n$$
$$\sum_{p=1}^{q} \phi_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} - R_{1j} = 0 \qquad j = 1, \dots, n$$
$$\sum_{r=1}^{s} u_r^1 y_{rj}^1 - \sum_{p=1}^{q} \omega_p z_{pj} - R_{21j} = 0 \qquad j = 1, \dots, n$$
$$\sum_{r=1}^{s'} u_r^2 y_{rj}^2 + \sum_{p=1}^{q} (\omega_p - \phi_p)\, z_{pj} - R_{22j} = 0 \qquad j = 1, \dots, n$$
$$\sum_{j=1}^{n} \left(R_{1j} + R_{21j} + R_{22j}\right) = R \qquad R_{1j}, R_{21j}, R_{22j} \geq \epsilon \quad j = 1, \dots, n$$
$$u_r, \phi_p, v_i \geq 0 \ , \quad r = 1, \dots, s \quad , \quad p = 1, \dots, q \quad , \quad i = 1, \dots, m (21)$$

It's worth noting that the model's limitations are established by the total standard deviation and system (15). (21).

$$\left|\alpha_j R - R_{1j}\right| - \alpha_j R + R_{1j} = 2c_{2j} \quad , \quad \left|\alpha_j R - R_{1j}\right| + \alpha_j R - R_{1j} = 2c_{1j}$$
$$\left|\beta_j R - R_{21j}\right| - \beta_j R + R_{21j} = 2d_{2j} \quad , \quad \left|\beta_j R - R_{21j}\right| + \beta_j R - R_{21j} = 2d_{1j} (22)$$

$|\gamma_j R - R_{22j}| - \gamma_j R + R_{22j} = 2e_{2j}$, $|\gamma_j R - R_{22j}| + \gamma_j R - R_{22j} = 2e_{1j}$ Since the model is non-linear, the following can be stated:

Model (21) is expressed as:

$$\begin{aligned}
&\underset{k=1,\dots,n}{Min\,Max}\,(c_{1k}+c_{2k}+d_{1k}+d_{2k}+e_{1k}+e_{2k})\\
s.t\quad &\alpha_j R - R_{1j} = c_{1j} - c_{2j} \qquad j=1,\dots,n\\
&\beta_j R - R_{21j} = d_{1j} - d_{2j} \qquad j=1,\dots,n\\
&\gamma_j R - R_{22j} = e_{1j} - e_{2j} \qquad j=1,\dots,n\\
&\sum_{p=1}^{q} \phi_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} - R_{1j} = 0 \qquad j=1,\dots,n\\
&\sum_{r=1}^{s} u_r^1 y_{rj}^1 - \sum_{p=1}^{q} \omega_p z_{pj} - R_{21j} = 0 \qquad j=1,\dots,n\\
&\sum_{r=1}^{s'} u_r^2 y_{rj}^2 + \sum_{p=1}^{q} (\omega_p - \phi_p)\, z_{pj} - R_{22j} = 0 \qquad j=1,\dots,n\\
&\sum_{j=1}^{n} \left(R_{1j} + R_{21j} + R_{22j}\right) = R \qquad R_{1j}, R_{21j}, R_{22j} \geq \epsilon \qquad j=1,\dots,n\\
&u_r, \phi_p, v_i \geq 0, \qquad r=1,\dots,s \quad , \quad p=1,\dots,q \quad , \quad i=1,\dots,m \qquad (23)
\end{aligned}$$

The model includes the min-max objective function, system restrictions (15), and standard deviation restrictions (23). The formula $\underset{k=1,\dots,n}{Max}(c_{1k}+c_{2k}+d_{1k}+d_{2k}+e_{1k}+e_{2k})$ determines the maximal deviation of effective allocation with size-based cost allocation across DMUs. Mode (23) is able to minimize the deviation DMUk. In terms of resource utilization and result production, it has the highest cost variation compared to all other DMUs, as well as the fixed-cost allocation plan.

4. Practical example

In this section, we deal with a fixed cost allocation in a real-world example using the model proposed in this article. Data is obtained from one of the geographical regions of Tehran. The goal is the fixed cost allocation for these branches of commercial banks in Iran. The fixed cost allocation to 37 branches is of interest. The system of these branches is in the network form, which consists of two inputs (m=2), one intermediate output (q=1), and three final outputs. One final output is considered for the first subphase (s=1), and two final outputs are considered for the second subphase (s=2). Two considered inputs are staff privilege and paid interest. Staff privilege is the costs the bank pays for its staff, including salary, mission, and benefits. The paid interest is the costs paid for the interests of deposits. The output of the intermediate stage is the sum of the four deposits, which is the collection of four major deposits of bank resources, including time deposit, demand deposit, saving account, and checking account. The system outputs in the first and second subphases are granted facilities and received commission, respectively. Banks offer the cash they receive in the form of deposits as facilities to natural and legal entities. One of the main sources of revenue for banks is the benefit resulted from granted facilities. This interest is gained from facilities granted to other banks and activities like this. Other sources of revenue for banks include the received charges, which is the nominal costs for setting up and holding different accounts of minor transactions services for retail and business customers. For example, the interest from the transfer of customers' funds, the charge for marketing securities, the interest gained from financial advising, and other revenues. The data for input, intermediate, and output indices for 37 bank branches are listed in the table (1).

Table (1): The input, intermediate, and output data of 37 bank branches

	Input1(I)	Input2(I)	Intermediate(Z)	Output1(O1)	Output2(O2)	Output3(O2)
1	57.16	7700462917	3.38858E+11	4.07492E+11	4433589964	1865081618
2	33.11	6582232881	3.95389E+11	5.40963E+11	4113085379	5316468446
3	14.91	3715848251	3.65318E+11	2.08045E+11	379200138	2626670580
4	24.68	5805036011	2.61669E+11	2.23048E+11	549186505	3223602333
5	31.45	13298332581	3.38697E+11	4.18272E+11	2630105133	1271257096
6	24.11	10879748209	6.48687E+11	1.70403E+11	2894197123	345215197
7	57.47	8207589534	2.52014E+11	4.74686E+11	16761736021	1031476939
8	32.31	9990341554	3.8833E+11	6.35852E+11	4873553145	2876273049
9	81.18	14429599187	4.07216E+11	1.31729E+12	13724803958	4741183069
10	56.47	19709435214	1.18397E+12	1.28233E+12	28895391222	13853020949
11	30.37	21721578702	4.64214E+11	7.30391E+11	31889276043	202986136

	Input1(I)	Input2(I)	Intermediate(Z)	Output1(O1)	Output2(O2)	Output3(O2)
12	51.58	5590799793	2.69727E+11	5.77874E+11	6293415973	4178903208
13	58.4	57767217488	1.1756E+12	2.82347E+12	74871695475	5288325390
14	29.71	2925417361	4.18832E+11	1.66235E+11	210068820	509255781
15	63.77	63744797380	1.27363E+12	4.51146E+12	2.08535E+11	2780450807
16	54.87	29533938975	6.30627E+11	3.24066E+11	1937989519	239142925
17	88.97	1.14743E+11	5.32808E+12	7.19824E+12	16706234229	1.54574E+11
18	44.35	54283998889	1.29976E+12	2.49876E+12	55369107239	4165383922
19	52.94	21644167552	1.0944E+12	1.24657E+12	2024085800	2753427275
20	46.99	66342159905	1.38868E+12	5.19914E+11	9323281997	2552605575
21	34.54	28067867398	1.0078E+12	6.05693E+11	3441854552	1483351227
22	42.26	9947145354	9.56524E+11	3.243E+11	5442078512	1774702342
23	48.68	58145106414	1.4478E+12	7.39077E+11	2171797688	955483463
24	55.73	19856358108	4.66381E+11	1.85578E+11	10685264563	4047671147
25	43.79	23191220929	5.5353E+11	2.18192E+12	7083379475	5010638944
26	21.44	23481720667	7.04444E+11	1.18388E+11	950601578	1170301759
27	17.68	12161248003	3.82158E+11	1.64462E+12	1753425	2622049198
28	42.78	15143739120	1.07607E+12	1.343E+12	50758250299	1542325695
29	33.24	1.06811E+11	1.66251E+12	2.87489E+12	541505628	86063470
30	23.35	3296967069	81738662033	82713558809	696437541	168213750
31	39.56	10094268507	4.09752E+11	1.74797E+12	21468687150	1679894635
32	33.55	45925290055	7.43058E+11	4.54772E+12	3851404	162047528
33	42.25	12343401251	2.65298E+11	3.35945E+11	9518569517	3476357041
34	38.94	4539574801	1.7796E+11	6.40356E+11	10140870618	953300702
35	37.08	34081801322	1.48267E+12	8.63924E+11	988372823	1887869477
36	35.68	16649315128	1.33777E+12	9.4794E+11	19402890832	2559354899
37	18.83	19010314739	4.30575E+11	4.41684E+11	10615620539	720914682

In the following of the model (23) proposed in this article, we deal with the fixed cost allocation for bank branches. In this stage, the cost allocated for each phase and efficiency of each stage and total efficiency of calculation and its results are shown in the table (2).

Table (2): Fixed cost allocation for bank branches

R1	R21	R22	e_1	e_2^1	e_2^2
2.16356	2.94260	3.63445	1.00000	1.00000	1.00000
3.66964	3.98325	4.24078	1.00000	1.00000	1.00000
3.92524	1.23104	3.91825	1.00000	1.00000	1.00000
2.33421	1.50320	2.80655	1.00000	1.00000	1.00000
3.03821	3.03332	3.63272	1.00000	1.00000	1.00000
7.05061	0.51530	6.95755	1.00000	1.00000	1.00000
1.09902	3.62916	2.70300	1.00000	1.00000	1.00000
3.61125	4.78974	4.16507	1.00000	1.00000	1.00000
2.17418	10.48334	4.36763	1.00000	1.00000	1.00000
12.44363	9.09408	12.69877	1.00000	1.00000	1.00000
4.59835	5.47629	4.97897	1.00000	1.00000	1.00000
1.51482	4.47037	2.89298	1.00000	1.00000	1.00000
12.27624	22.04278	12.60900	1.00000	1.00000	1.00000
4.07008	0.80458	4.49222	1.00000	1.00000	1.00000
13.28286	36.07410	13.66043	1.00000	1.00000	1.00000

R1	R21	R22	e_1	e_2^1	e_2^2
5.78263	1.83068	6.76384	1.00000	1.00000	1.00000
61.62972	52.90817	57.14676	1.00000	1.00000	1.00000
14.26213	19.14189	13.94068	1.00000	1.00000	1.00000
11.47694	8.92026	11.73808	1.00000	1.00000	1.00000
15.25128	2.40529	14.89440	1.00000	1.00000	1.00000
11.05330	3.66267	10.80924	1.00000	1.00000	1.00000
10.16778	1.37289	10.25928	1.00000	1.00000	1.00000
15.91115	4.16162	15.52850	1.00000	1.00000	1.00000
3.75997	0.89987	5.00221	1.00000	1.00000	1.00000
5.22475	17.53495	5.93693	1.00000	1.00000	1.00000
7.81833	0.00001	7.55557	1.00000	1.00000	1.00000
4.03517	13.26642	4.09887	1.00000	1.00000	1.00000
11.60089	9.75559	11.54148	1.00000	1.00000	1.00000
19.04338	21.78752	17.83139	1.00000	1.00000	1.00000
0.19586	0.57902	0.87669	1.00000	1.00000	1.00000
3.62404	14.09505	4.39483	1.00000	1.00000	1.00000
7.87406	37.12697	7.96973	1.00000	1.00000	1.00000
1.77918	2.44578	2.84548	1.00000	1.00000	1.00000
0.83208	5.12433	1.90872	1.00000	1.00000	1.00000
16.72986	5.16044	15.90250	1.00000	1.00000	1.00000
15.01905	6.07012	14.34836	1.00000	1.00000	1.00000
4.58356	3.10023	4.61817	1.00000	1.00000	1.00000

According to the model proposed in this article, fixed cost allocation is based on the size of DMUs and the magnitude of consumed inputs and produced output which size of each DMU is adjusted with two vector norm (X, Z, Y) of that DMU. By considering this issue, we conclude that DMU_{17} received the maximum cost allocation based on the input and output values in each of the three proposed allocations. DMU_{29}, DMU_{15}, DMU_{18}, DMU_{32}, and DMU_{25} are units that cost allocation of them is high. DMU_{30} received the least cost allocation in all three proposed allocations based on the input and output values. DMU_4, DMU_7, DMU_{24}, DMU_{33}, DMU_{14}, and DMU_{34} are units that the minimum cost allocation is conducted in them. By considering the efficiency columns of the table (2) (which are columns 5 to 7), they all became equal to one, and this approves theorem 1.

5. Conclusion

The main problem facing the decision-makers in big organizations is the allocation of the cost of a common cost across a series of entities. The DEA approach is most commonly used to solve the problem of fixed cost allocation. However, techniques capable of addressing this issue in a network setting are uncommon. In the current study, the following approach is used to redistribute fixed costs among all DMUs in a combined network configuration (i.e., a series and parallel network). The relative efficiency of the integrated network is first determined using allocated expenses. Next, an effective allocation set is presented that is, using a set of common weights, capable of achieving a synchronized efficiency for all stages. In the present case, fixed cost allocations lead to all DMUs becoming efficient. The operating size of units is determined in order to optimize the allocation strategy which is concerned with the constant cost allocation in relation to the inputs and outputs generated by DMUs. This means that DMUs are assigned based on their size rather than their capacity. The allocation plan aims to decrease the deviation between sized-based allocation and efficient allocation to the minimum.

Despite the logical structure/design/framework of this study, it was still faced with a few limitations. Although the measurement of the unit operation size and the optimization of the allocation plan were carried out through a practical approach, its optimality and acceptability are not certain. Simultaneously, techniques based on the concept of performance maximization may result in a substantial allocation to DMUs and highly efficient scores, penalizing DMUs even more effectively. As in this study, the operation size is taken into account, this would not be problematic in the proposed method.

The proposed method was implemented using the data of 37 branches of one of the banks of Iran and the fixed cost was allocated based on the aggregate of inputs and outputs of the bank branches and all DMUs and subphases became efficient after fixed cost allocation.

The following are the general aspects of this study. First, the idea of considering the operation size of DMUs is worthy of attention as it may even alter the outcome of allocation. It is, therefore, recommended that future researchers conduct further studies on the subject. Second, this method is remarkable in fully dividing the allocated costs from initial efficiency, irrespective of the DEA method being concerned with performance. Third, such approaches are applicable to different network structures as substantial changes have occurred in the two-stage structures. Fixed-cost allocation for networks with undesirable factors and parallel production processes are two further uses.

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